MINERVA

Temperature evolution in a pure electron plasma

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Thank you

Anja Hahn I LOVE YOU

Marianne, Jürg, Philippe and Christoph Zwahlen For standing by me.

Reini Brunner You kept me on the ground – thank you. I love you

Prof. C.Amsler Physics is fun. Thank you for giving me a chance

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Introduction

From April until December I have studied the energy distribution of a pure electron plasma. The measurements were done on MINERVA at the European Laboratory of Particle Physics CERN. The energy distribution is one of the key properties of a plasma. With it, the plasma size and a wave analysis the dynamics of the plasma are fully determined.
The synchrotron radiation of the electrons in a magnetic field should cause the plasma to loose a lot of (thermal) energy and thus cool down. My task was get a procedure for the temperature evaluation working and test the synchrotron cooling mechanism. The whole experiment was done as a test for ATHENA. ATHENA is one of the three future experiment at the antiproton-deccelerator at CERN. The aim of ATHENA is to produce antihydrogen study its spectrum and compare the result with a hydrogen reference measured on the same machine. As projected this measurement will be the "highest-precision" CPT-test so far – relative world↔antiworld asymmetries of 10^{-18} should be observable.
In the whole ATHENA it is important to have very low temperatures for three reasons:

1. The spontaneous recombination rate $e^+ + \overline{p} \rightarrow \overline{H}$ decreases rapidly with increasing energies.

2. The capturing potential for neutral "anti-atoms" in a quadrupol field is small– hot atoms can not be held

3. The precision of the experiment can only be achieved if the lasers do not encounter fast moving objects. Any optical Doppler-effect should be negligible.

There are two plasmas involved in ATHENA. The first is a positron plasma. It is the first half of the antihydrogen. The second is an electron plasma. It is not directly used but serves as a decelerator for the antiprotons. The temperature of the electron plasma is the average energy of the antiprotons.
All this makes it clear that temperature measurement is a crucial diagnostic for ATHENA.

Chapter 1

Electrodynamics of plasmas

1.1 Typical fields

1.1.1 The solenoid field

I will only say a few words about the solenidal field. The magnetic field within a long coil with length l and N windings is given by:

$$\vec{B} = \mu_0 \frac{N}{l} I \equiv A_1 I \cdot \vec{e}_z \tag{1.1}$$

This is in true as long as you look well inside the magnet (no boundary effects) and the windings are regular. For the MINERVA system A_1 is known to be $A_1 = 0.01718(5)\frac{T}{A}$. This constant was measured in the factory.

1.1.2 The field in a cylinder

In this section I will present, how the electric field and potential of a cylinder with applied voltage looks like. The relevance of this situation for Minerva becomes clear, when you consider, that the electrodes in my trap are all cylindrical. The solution to this problem is “straight forward”: The potential obeys the Laplace-equation. So solving the problem means solving that differential equation for a given set of boundary values. I'll start with a ϕ-symmetrical potential and the Laplace-equation in cylindrical coordinates.

$$\frac{\partial^2 V}{\partial \rho^2} + \frac{1}{\rho}\frac{\partial V}{\partial \rho} + \frac{\partial^2 V}{\partial z^2} = 0 \tag{1.2}$$

$$V(\rho, z) \equiv f(\rho) g(z) \tag{1.3}$$

which leads to:

$$\frac{1}{f}\frac{\partial^2 f}{\partial\rho^2} + \frac{1}{f\rho}\frac{\partial f}{\partial\rho} = -\frac{1}{g}\frac{\partial^2 g}{\partial z^2} \equiv \alpha^2 \tag{1.4}$$

Setting both the ρ- and the z-part to match a constant is just an attempt, but seems reasonable considering the fact, that the Laplace-equation has to be valid for every point in space. Assuming a positive value for these terms will prove to be the right choice but is of course not a priori evident.
If I ask for a solution, that is symmetrical in z and for a potential of zero at the end of the electrode ($z \equiv \pm z_0 = \frac{l}{2}$), I immediately get:

$$g(z) \propto \cos(\alpha_n z) \tag{1.5}$$

with $\alpha_n = \frac{(2n+1)\pi}{2z_0}$. Note that the z-dependent equation gives a restriction for possible values of α.
The equation for ρ is called the "Bessel equation" and has the following solution:

$$\begin{aligned} \frac{d^2 f}{d\rho^2} + \frac{1}{\rho}\frac{df}{d\rho} + (\imath\alpha)^2 &= 0 \\ \Rightarrow \qquad f(\rho) &= f_0 \cdot J_0(i\alpha\rho) \end{aligned} \tag{1.6}$$

J_0 stands for the Bessel function of order zero. Now I will combine the solutions and ask for a linear combination that fulfills the boundary condition V(R) = -1 $\forall z \in [-z_0, z_0]$.
This gives me the solution: Decomposition of potential

$$\begin{aligned} V(\rho, z) &= \sum_{n=0}^{\infty} c_n J_0(i\alpha_n\rho)\cos(\alpha_n z) \\ c_n &= \frac{4(-1)^{n+1}}{(2n+1)\pi J_0(i\alpha_n R)} \end{aligned} \tag{1.7}$$

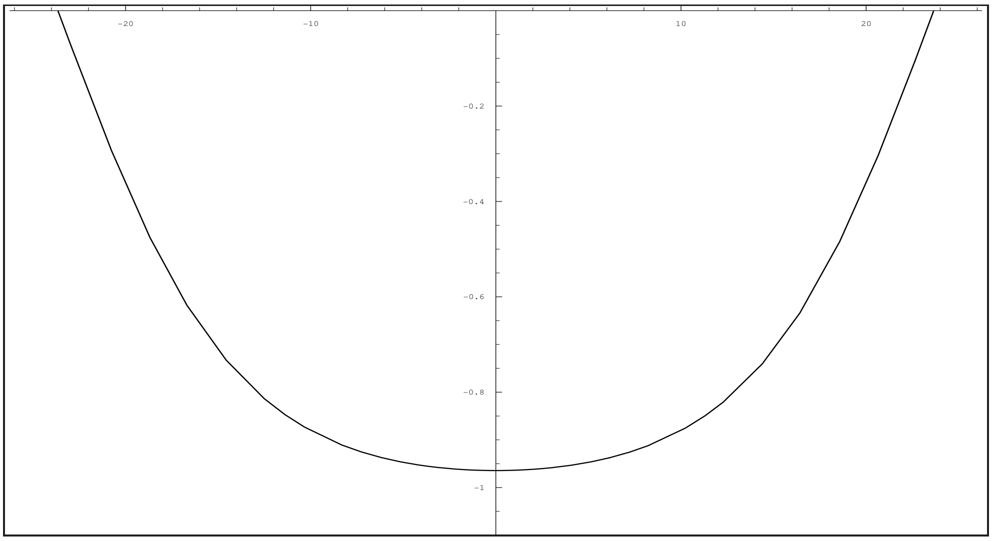


Figure 1.1: Electrostatic potential along the central axis. The dimensions are taken from the real MINERVA electrode geometry.

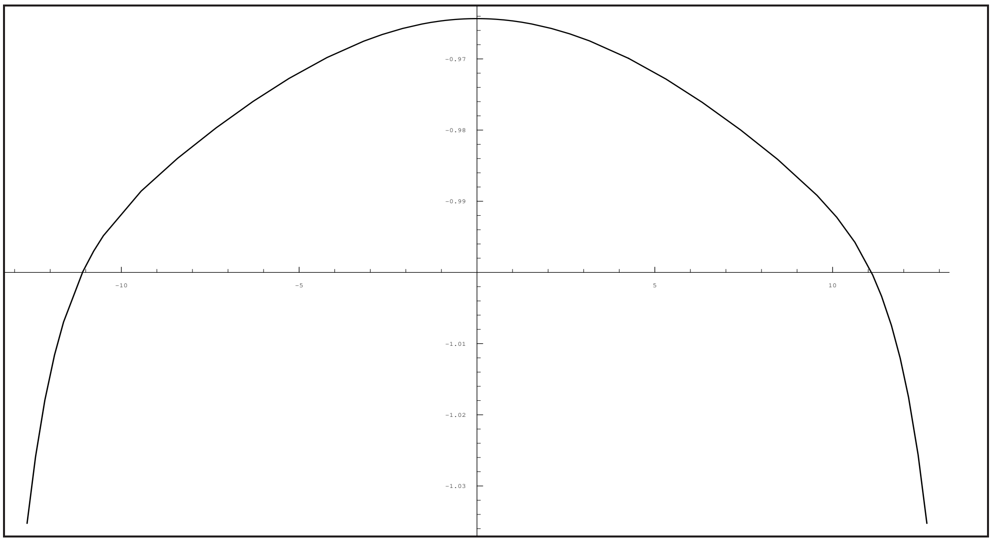


Figure 1.2: Electrostatic potential in radial direction from the center of the electrode. The dimensions are taken from the real MINERVA electrode geometry.

1.2 Motions of particles in electromagnetic fields

1.2.1 Introduction

A nonneutral plasma consists of a lot of charged particles. Each one of them fulfills:

$$m\frac{d\vec{v}}{dt} = q(\vec{E} + \vec{v} \times \vec{B}) \tag{1.8}$$

The motion of a particle is fully determined by this equation. The only problem is, that the fields strongly depend on positions and velocities of all the other particles. Solving this multiparticle system is virtually impossible. Nevertheless one can learn a lot by examining movements of single particles, because this gives a lot of qualitative information on how the trajectories look like.

nonrelativistic equation of motion

1.2.2 Movement in a homogeneous magnetic field

Let's start with the movement of a nonrelativistic particle in a constant, homogeneous magnetic field $\vec{B}(\vec{R}) = B_z \vec{e_z}$ The nonrelativistic equation of motion simplifies to:

$$m\frac{dv_x}{dt} = qv_y B_z \tag{1.9}$$
$$m\frac{dv_y}{dt} = -qv_x B_z \tag{1.10}$$
$$m\frac{dv_z}{dt} = 0 \tag{1.11}$$

Differenting for time and inserting the two coupled equations gives the following result.

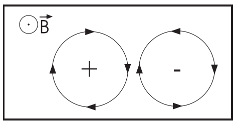
$$v_x = v_\perp \sin(\Omega t) \tag{1.12}$$
$$v_y = v_\perp \cos(\Omega t) \tag{1.13}$$
$$v_z = v_z \tag{1.14}$$

With $\Omega = \frac{qB_z}{m}$. What kind of orbits will one see in this homogeneous field? Any movement parallel to $\vec{B}$ will not be affected, so particles can move parallel to the magnetic field with a constant velocity. If a particle has a velocity component which is not parallel to $\vec{B}$ then it will start flying a circle in the plain perpendicular to $\vec{B}$. The radius and frequency can be calculated using the condition for a circular movement.

gyromotion

Orbits look like a helix

$$R = \frac{mv_\perp}{qB_z} = \frac{\sqrt{2m}}{qB_z}\sqrt{E_\perp} \tag{1.15}$$
$$\nu_{cyclotron} = \nu_c = \frac{1}{2\pi}\frac{eB_z}{m} \tag{1.16}$$



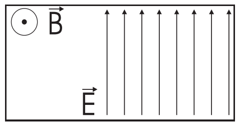
By simply looking at the direction in which the Lorentz force is pointing, one finds that negatively charged particles go around in the positive direction, while positive ones turn in the opposite direction. The two expressions "gyration" and "cyclotron motion" can be used synonymous.

	B = 2T	B = 1T	B = 0.5T
E[eV]	R[μm]	R[μm]	R[μm]
$\frac{4}{11600}$	0.03	0.06	0.12
0.001	0.05	0.1	0.2
0.01	0.15	0.3	0.6
0.1	0.5	1	2
0.5	1.2	2.4	4.8
1	1.7	3.4	6.8
2	2.4	4.8	9.6
4	3.4	6.7	13.4
10	5.4	10.7	21.4

Table 1.1: Gyroradii for different magnetic field strengths and perpendicular kinetic energies. The lowest energy corresponds to liquid helium.

The gyroradius of electrons in a sufficiently high magnetic field is very small compared to other dimensions like trap dimensions or axial spielraum. It is interesting to see how well a single particle is prevented from moving across a magnetic field. Just for fun you can calculate the electric field strength needed for a force equivalent to the Lorentz-force of an electron with $E_{\perp} = 1eV$ in a $1T$ field. The acceleration resulting hereof is the dominant acceleration an electron sees in MINERVA

1.2.3 Homogeneous $\vec{B}$ -field with a perpendicular $\vec{E}$ -field



In addition to the magnetic field I would now also like to have a constant electric field $\vec{E}$. I split this field into

$$\vec{E} = \vec{E_\perp} + \vec{E_\parallel} \tag{1.17}$$

Of course the parallel and orthogonal are to be understood with respect to the magnetic field. I will again have to solve the nonrelativistic equation of motion (equation 1.8) as introduced in 1.2.1. The $\vec{E_\parallel}$ doesn't interest me since the particle can move along the magnetic field lines anyway. So this parallel component just gives a constant acceleration in the direction of the magnetic field.
To find out, what the $\vec{E_\perp}$ term does I start with the following parametrization:

$$\vec{v_\perp} \equiv \frac{\vec{E_\perp} \times \vec{B}}{B^2} + \vec{u} \tag{1.18}$$

Then you get (omitting the parallel part):

$$\frac{d\vec{u}}{dt} \underbrace{=}_{def} \frac{d\vec{v_\perp}}{dt} \underbrace{=}_{\text{equation of motion}} \frac{q}{m}\frac{\vec{E_\perp} \times \vec{B}}{B^2} \times \vec{B} + \frac{q}{m}\vec{u} \times \vec{B} + \frac{q}{m}\vec{E_\perp} \tag{1.19}$$

This does not look as if we gained anything. But using $\|\vec{A} \times \vec{B}\| = \|\vec{A}\|\,\|\vec{B}\|\sin(\alpha)$, our orthogonal setup and the 'write hand rule' equation 1.19 simplifies to:

$$\frac{d\vec{u}}{dt} = -\frac{q}{m}\frac{\vec{E_\perp} B^2}{B^2} + \frac{q}{m}\vec{E_\perp} + \frac{q}{m}\vec{u} \times \vec{B} \tag{1.20}$$

$$= \frac{q}{m}\vec{u} \times \vec{B} \tag{1.21}$$

which I of course don't have to solve again, since it's the same equation I had for only a magnetic field.

So what happens is,that a particle still turns in the circles but now also has a constant "drift" $\frac{\vec{E_\perp} \times \vec{B}}{B^2}$. Quite surprisingly this drift does not depend on the charge of the particle and is perpendicular to the electric field.

Constant drift across $\vec{B}$ and perpendicular to $\vec{E}$

This situation can of be generalized for any homogeneous force, giving a drift velocity of

$$\vec{v}_{\text{drift}} = \frac{1}{q}\frac{\vec{F_\perp} \times \vec{B}}{B^2} \tag{1.22}$$

Because the magnetic field in MINERVA is almost parallel to the gravitational force, this field does not give a $\vec{G} \times \vec{B}$ drift.

1.2.4 Generalization of the $\vec{E} \times \vec{B}$ drift–guiding center theory

The previous section showed two remarkable things.

- The motion could be split up into the gyromotion, that is a rotation, and a (constant) drift motion, in that case called $\vec{E} \times \vec{B}$ -drift. This orbit looks as if the particle is still circling around a central point (called the center) but in addition the center is moving as well.
- The direction in which the particle is moved is perpendicular, to the direction of the field lines.

Omitting the gyromotion looks like a nice approach because as shown in section 1.2.2 the gyroradii are generally very small compared to trap sizes or plasma lengths and thus don't interest me for the time being as long as I know the orbit of the center.

"Explanation" for the Drift across $\vec{E}$

In order to say something about the validity of the guiding center theory, I would like to understand why the drift is perpendicular to the $\vec{E}$ -field.

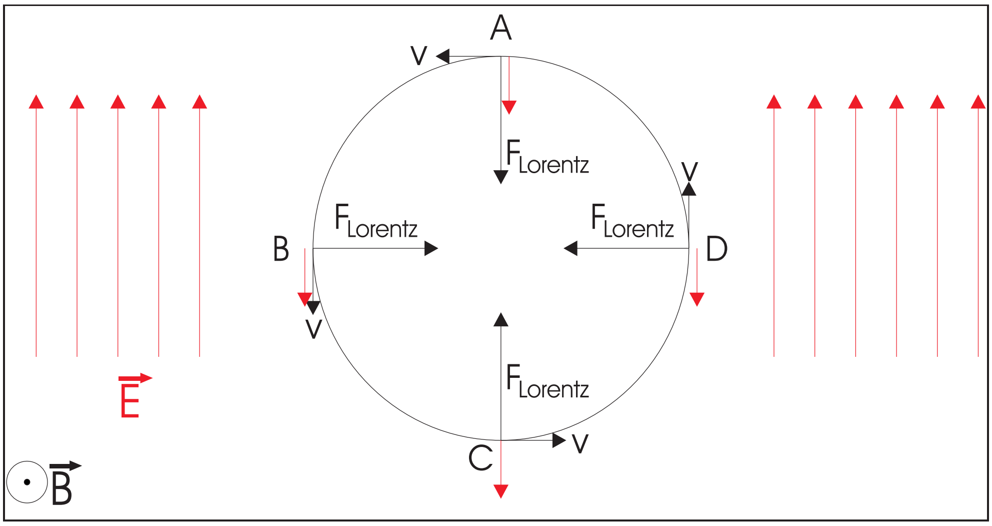


Figure 1.3: The additional force can be considered constant and thus acts against or in favor of the Lorentz-force.

Fig.(1.3) shows a particle in the situation of 1.2.3. The figure is an unperturbated particle, gyrating around the standing center. The arrows show the direction of the force stemming from the electric field. As one sees, this force pulls in the same direction as the Lorentz-force at A, while it acts against the Lorentz-force at B. Considering equation (1.15) one finds:

Point A: The electric force enlarges the Lorentz-force. The gyroradius becomes smaller

Point B: The electric field accelerates the particle, forcing it onto a larger radius

Point C: The electric force works against the Lorentz-force and the velocity reaches it's maximum. So does the gyroradius

Point D: The particle is decelerated and thus has a smaller gyroradius

The reason I still use the word gyroradius is because the Lorentz-force always wants to force the particle onto a circular motion. The typical geometrical forms that show this behaviour are called "cycloids".
Using our knowledge of those cycloids (remember, that they describe the orbit of a point on a rotating wheel) we can understand the fact, that the drift is perpendicular to the force. In the example of the wheel the force would of course be the gravitation.

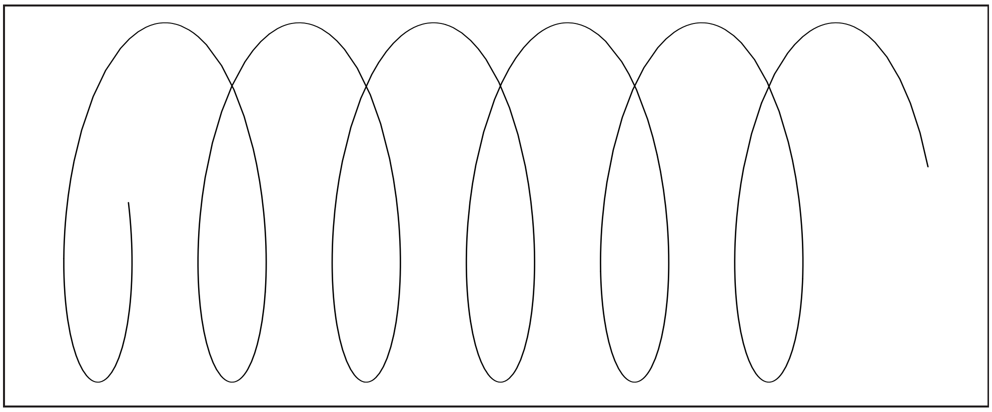


Figure 1.4: A cycloid is the orbit of a fixed point on a unrolling wheel

Does this idea of a particle that spins because it's gyroradius varies from place to place make sense? In general no. Of course one always wants a magnetic field, because you don't see any gyromotion without the magnetic field. A fairly safe condition is denoted by the following equation:

$$\frac{\max_{x\in D, y\in D} \mid \vec{F(x)} - \vec{F(y)} \mid}{\min_{x\in D} \vec{F(x)}} \ll 1 \quad \forall D(s) := \left\{ x \in \mathbf{R}^3 : \mid x - s \mid < R_{\text{gyro}} \right\} \qquad (1.23)$$

All this condition does, is reassure, that changes in direction and strength of the force stemming from the perturbation are very small. This way we can locally treat the force as constant and thus locally apply the rules for a homogeneous constant field. (compare section: 1.2.3)

Orbits locally look like cycloids

Thanks to this approximation, we get a simple and intuitive characterization of possible orbits. All trajectories of particles in a situation dominated by a $\vec{B}$ -field of high uniformity locally look like cycloids.
Lets look at a cycloid. (See Fig.1.4) The mathematical formula that describes a cycloid is:

$$\begin{pmatrix} x \\ y \end{pmatrix}(s) = \begin{pmatrix} ms + \sin(\omega s) \\ \cos(\omega s) \end{pmatrix} \tag{1.24}$$

Particles move across the perturbating fieldlines

The movement of the rotational center is parallel to a tangent at the place with the smallest curvature (That is equivalent to the largest radius). Transferring this to our orbits, this means: The direction of the guiding center is in the direction of the speed of the particle, at the place, where its gyroradius is locally maximal. This is the case, if the perturbating force points in the opposite direction of the Lorentz force.

Application of the guiding center: unidirectional $\vec{B}$ -field with field gradient

With the knowledge of the preceding subsection, we can now quite easily analyze a couple of field situations. The first assumes, that the magnetic field is now not constant anymore, while still timeindpendent and unidirectional.
The nondisappearing fieldgradient (of for instance B_z) gives an additional Lorentz force that once again results in a locally varying cyclotron radius. Where is the place, where the curvature is the lowest? The answer is easy, it's where the additional Lorentz force opposes to the "normal" one. The resulting drift of the particle will thus be perpendicular to the magnetic field gradient.
The drift velocity can quite easily be calculated. I'll do this for a $\vec{B}$ -field $\vec{B} = B(x)\vec{e}_z$. Obviously then the gradient will have the form

$$\operatorname{grad}\vec{B} = \frac{\partial B}{\partial x}\vec{e}_x \tag{1.25}$$

The drift velocity is parallel to $\vec{e}_y$. In order to evaluate the speed, I will calculate the average force and then reduce this equation to the one for the $\vec{E} \times \vec{B}$ drift. The force for the particle is given by:

$$\begin{aligned} \vec{F} &= q\vec{v} \times \vec{B} \\ F_y &= -qv_x B_z \\ F_x &= qv_y B_z \end{aligned} \tag{1.26}$$

Assuming all conditions for perturbation theory (the requirement for drift theory allows me to do so) are fulfilled I insert the unperturbated orbit in order to get the velocities.

$$\begin{aligned} F_y &= -qv_\perp \sin(\omega t)(B(x_0 - \frac{v_\perp}{\omega}\cos(\omega t))) \\ &= -qv_\perp \sin(\omega t)(B_0 - \frac{v_\perp}{\omega}\cos(\omega t)\frac{\partial B}{\partial x}) \qquad (1.27) \\ F_x &\overset{analog}{=} qv_\perp \cos(\omega t)(B_0 - \frac{v_\perp}{\omega}\cos(\omega t)\frac{\partial B}{\partial x}) \qquad (1.28) \end{aligned}$$

The temporal averages are calculated by integrating over one period and dividing through the period. This leads to:

$$\begin{aligned} \langle F_y \rangle &= 0 \qquad (1.29) \\ \langle F_x \rangle &= -\frac{q}{\omega} v_\perp^2 \frac{\partial B}{\partial x} \cdot \frac{1}{2} \qquad (1.30) \end{aligned}$$

Now I have brought this into the form of the $\vec{F} \times \vec{B}$ situation, giving me a drift velocity equal to:

$$\vec{v}_{\text{drift}} = \frac{1}{q}\frac{\langle \vec{F} \times \vec{B} \rangle}{B^2} = -(\frac{v_\perp^2}{2\omega})\frac{\nabla \vec{B} \times \vec{B}}{B^2} \qquad (1.31)$$

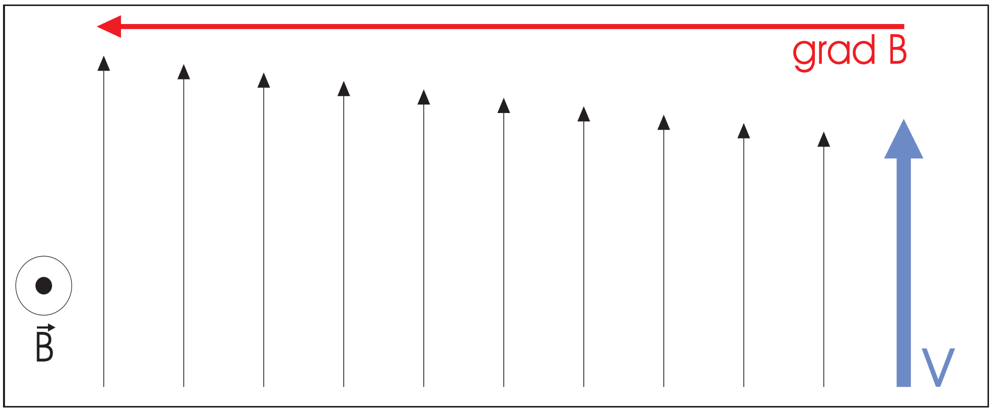


Figure 1.5: $\nabla\vec{B}$—drift. A magnetic field gradient provokes a drift perpendicular to the gradient.

1.3 The Minerva trap situation.

As next thing I will discuss what happens to a particle that moves in a Penning trap with cylindrical electrodes. An illustration of such a trap can be found on page ***, where I present the experimental setup of MINERVA . The field of such an electrode, for an applied voltage was described in 1.1.2. I will discuss the radial and the axial movement individually. Combining the to motions that generally don't decouple makes little sense, since I am mainly interested in a qualitative knowledge of possible motions. Also the situation will change as soon as I make the step from a single particle to a plasma. Nevertheless the mechanisms stay the same.

1.3.1 The radial movement

With the knowledge of the guiding drift center the description of the radial movement is greatly simplified. On any electron, that is off center, an electrostatic force will

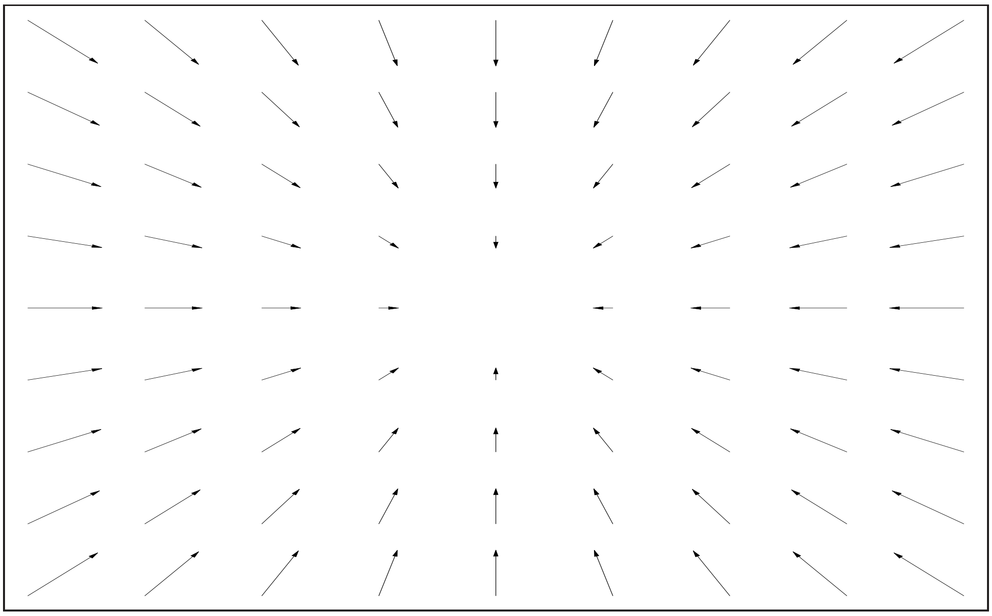


Figure 1.6: The radial force-field lines seen by an electron.

be acting. This force always points towards the radially inwards. Just due to the guiding drift theory, we can thus predict that the particle will start circling around the center, due to $\vec{E} \times \vec{B}$–drift.

Setting up the equation of motion is quiet straight forward.

$$\begin{aligned}\ddot{x} &= \frac{qB_z}{m}\dot{y} - \frac{a^2}{2}x \\ \ddot{y} &= -\frac{qB_z}{m}\dot{x} - \frac{a^2}{2}y \\ \frac{a^2}{2} &= \frac{q}{m}E = \frac{q}{m}\cdot(-\nabla V_{\text{electrode}})\end{aligned} \tag{1.32}$$

The velocity dependent term is just the Lorentz–force. The second term represents the extra force, due to the electrostatic potential applied on the electrode. The proportion to the position (x,y) guarantees a radially symmetric field. To prove this, just calculate the modulus of the force. In reality strength of the force is not given by a constant $\frac{a^2}{2}$ but since the motion will be circular, I can neglect this radial dependence. The solutions of the equations of motions for the initial conditions $x(0) = R_1 + R_2$, $y(0) = 0$,$v_x(0) = 0$,$v_y = -R_1\omega_1 - R_2\omega_2$ are given by:

$$\begin{aligned}x(t) &= R_c\cos(\omega_c t) + R_2\cos(at) \\ y(t) &= -R_c\sin(\omega_c t) - R_2\sin(at) \\ \omega_* &= \frac{\omega_c}{2} + \frac{1}{2}\sqrt{\omega_c^2 - 2a^2} \underbrace{\approx}_{a\ll\omega_c} \omega_c\end{aligned} \tag{1.33}$$

$$\omega_m = \frac{\omega_c}{2} - \frac{1}{2}\sqrt{\omega_c^2 - 2a^2} = \omega_c - \omega_* \ll \omega_c \tag{1.34}$$

The exact formulas for the new frequencies ω_* and ω_m are not important. The essential thing to know is, that the small rotation is fast (nearly the same frequency as the unperturbated cyclotron motion) and the large rotation is slow in the typical fields encountered at MINERVA . The typical region in which ω_m, the so called "magnetron motion", lies is about $10kHz$.

Mathematicians call these orbits hypercycloids. They look like a wheel rolling on the surface of an other wheel.

The additional rotation on a large radius is called magnetron motion.

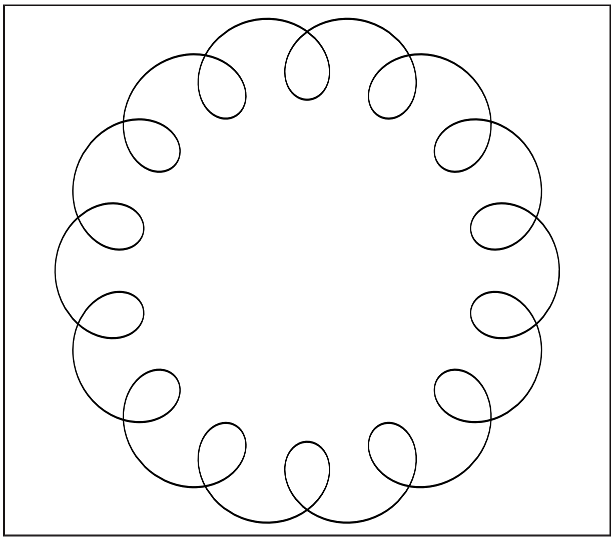


Figure 1.7: A typical hypercycloidical orbit of an off-centered electron. The gyro-oscillation is considered much faster than the magnetron-motion

The radius R_2 is given by the offset the particle has at the beginning of the motion. Afterwards it stays for long times. This offset is a macroscopical dimension. The particle will have this kind of offset, because the filament (see ***) is not well aligned with the trap and does not depend on the magnetic field. It is much bigger than the gyroradius. Forgetting the gyration leaves a circular motion.

1.3.2 The axial motion

The motion in direction of the magnetic field is given alone by the electric field of the electrodes. In first approximation a particle will feel a small force while it is away from the electrodes and a very big force as it reaches the electrode. This is easy to understand. The force is proportional to the gradient of the electrostatical potential, which I calculated in ***.

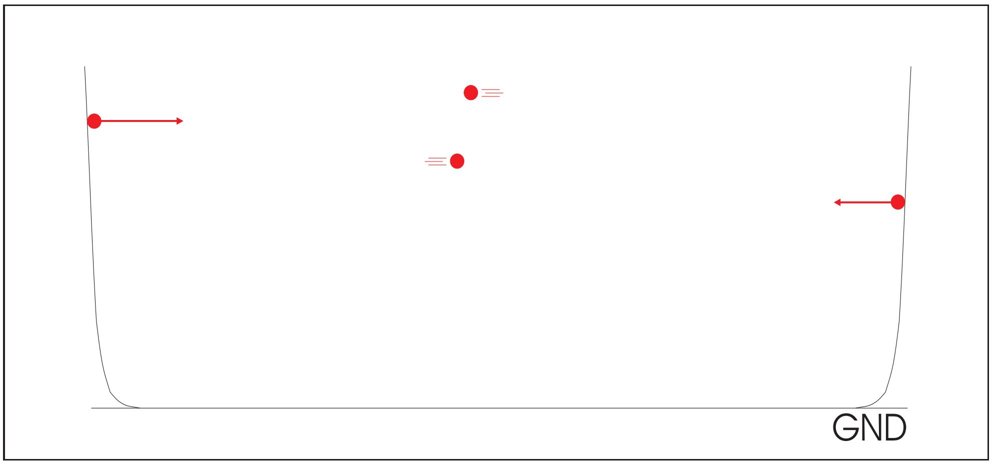


Figure 1.8: A simplified view of the potential in axial direction. A particle moves with nearly constant velocity between the electrodes and deflected very abruptly at the entrance of the electrodes

Using this simplified model makes it easy to describe the axial motion. If a particle starts in the middle of the trap with a kinetic energy E_{kin} it will just bounce to and fro. The bounce frequency is given by:

$$\nu_{\text{bounce}} = \frac{2s}{v} = \frac{2s}{\sqrt{\frac{2}{m}E}} \tag{1.35}$$

Here s is the distance between the two electrodes. For an electron with a kinetic energy of 1eV and a distance of 5cm the bounce frequency equals to about 6MHz. Especially with respect to the following section on radiation it is important to note that the average acceleration for the axial moment is rather small.

1.4 Radiated power of an accelerated charge

Here I would like to calculate the amount of power a particle emits if it gyrates in a constant $\vec{B}$ -field. This formula will be used later on to calculate the cooling rate of a plasma. According to [1] on pages 658–662 the power emitted by an accelerated particle is:

$$P = -\frac{dE}{dt} = \frac{2}{3}\frac{e^2}{c^3}|a|^2 \tag{1.36}$$

where a is the acceleration.
The next thing I have to do is calculate the acceleration for a gyrating particle. Using

the condition for circular movement $a = \omega^2 r$ and equations (***) I calculate:

$$a = \left(\frac{eB_z}{mc}\right)^2 \left(\frac{mv_\perp}{eB_z}\right) = \frac{eB}{mc}\sqrt{\frac{2}{m}E_\perp} = \omega_c v_\perp \tag{1.37}$$

which leads to:

$$\frac{dE}{dt} = -\frac{4}{3}\frac{e^4 B^2}{m^3 c^5} E_\perp \equiv -\alpha E_\perp \tag{1.38}$$

Which results in:

$$\begin{aligned} E(t) &= E_0 \cdot \exp(-\alpha t) = E_0 \cdot \exp(-\frac{t}{\tau_s}) \\ \alpha &\approx 3.87 \cdot 10^{-9} s^{-1} \cdot B_z^2[G] \end{aligned} \tag{1.39}$$

B[T]	B[G]	$\alpha[\frac{1}{s}]$	$\tau_s[s]$
0.1	1000	$3.87 \cdot 10^{-3}$	259
0.5	5000	$9.66 \cdot 10^{-2}$	10
1	10000	0.39	2.58
2	20000	1.55	0.645

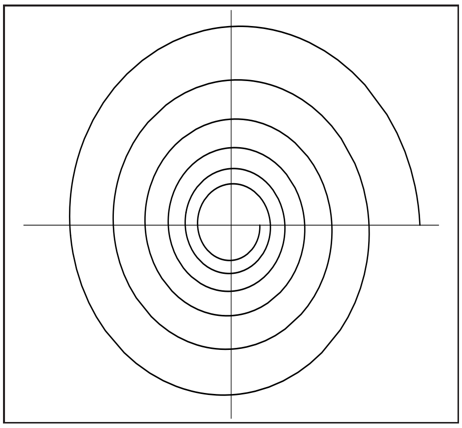


Figure 1.9: A gyrating and radiating particle rotates with constant angular velocity and exponentially decreasing radius.

Off course it is not clear, which acceleration makes the main contribution to the radiation losses. But comparing electrical forces $a = \frac{q}{m}E$ and the Lorentz-force

$a_l = \frac{q}{m} vB$ quickly resolves that the Lorentz-force is the dominant term for sufficiently high magnetic fields.
If the particle doesn't only move perpendicular to the magnetic field, the solution gets more complicated. This is due to the fact that the radiation is preferably in direction of the velocity. This means that not only will the rotation move to a smaller radius, but also the parallel motion will be slowed down.
The solution of this problem is difficult because the angular distribution of the emitted radiation depends on the speed of the particle. I would just like to mention two cases:

$\mathbf{v}_{\|} \ll \mathbf{v}_{\perp}$ Most of the kinetic energy is in the motion perpendicular to the $\vec{B}$ -field. Thus the acceleration and emitted power are large. The particle is slowed down very rapidly.

$\mathbf{v}_{\|} \gg \mathbf{v}_{\perp}$ The main part of the kinetic energy does not contribute to the deceleration. The particle is slowed down rather steadily. Particles that have a high percentage of "parallel" velocity remain fast for a longer time than those with high "perpendicular" speeds.

This will later on give rise to the concept of two different temperatures of a plasma. One perpendicular, that thermalizes quickly and one parallel to the magnetic field, that changes more slowly or not due to the synchrotron-radiation. Equation (***) gives an upper limit for the cooling speed of a plasma due to radiation cooling. It is interesting to note, that if one considers pure gyration (that is with no additional velocities) the cooling rate, is the same for any initial energy. This means, that an energy distribution essentially stays the same in form but merely gets squeezed together.
Obviously this can not be the whole cooling mechanism. Radiation losses would cause a plasma to cool down to 0K. The synchrotron radiation is however the main cooling mechanism for high temperature plasmas.

1.5 Interparticle processes — Collisions and scattering

Until now I have only looked at a single particle in a timeindependent electromagnetic field. The next step towards the plasma situation is looking at how particles interact with each other. In this section the number of particles will still be very small (≈ 2). All processes are assumed to be elastic (No chemistry involved)

1.5.1 Transfer of momentum

The dynamics of the scattering process is of course well defined for a given initial state. But as I have to average over a lot of possible scatterings anyway, I will not look at that now. I would like to understand how much momentum is transferred during a collision as a function of the scattering angle. Since all processes are elastic and the system is closed, both total momentum and energy are conserved. Sitting in the resting frame of particle 2, this results in:

$$\begin{aligned} m_1\vec{v}_1 &= m_1\vec{v}_1^* + m_2\vec{v}_2^* \\ m_1 v_1^2 &= m_1 {v_1^*}^2 + m_2 v_2^2 \end{aligned} \tag{1.40}$$

Figure (**) shows the three non–zero vectors of momentum. For the coordinates

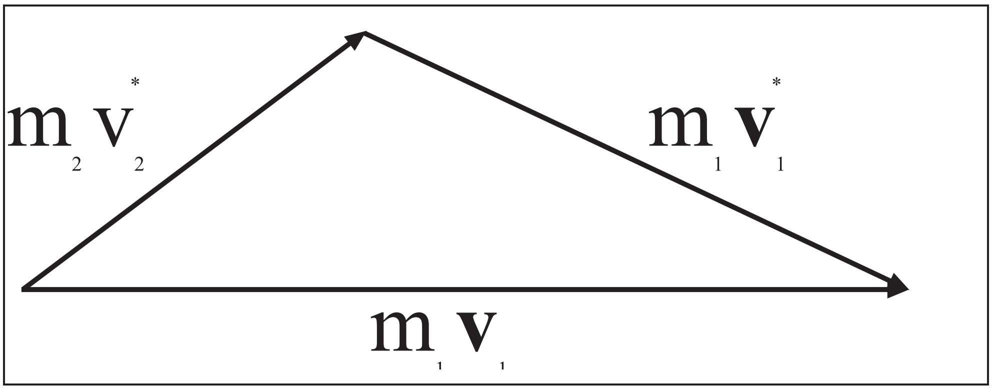


Figure 1.10: Conservation of momentum

(x,y) of the point P, that is the endpoint of $m\vec{v}_2^*$ I can write (this is just applying Pythagoras's law):

$$\begin{aligned} x^2 + y^2 &= m_2^2 {v_2^*}^2 \\ (m_1 v_1 - x)^2 + y^2 &= m_1^2 {v_1^*}^2 \end{aligned} \tag{1.41}$$

Inserting these equation into the equation for the conservation of energy (**) gives results in:

$$m_1 v_1^2 = \frac{1}{m_1}((m_1 v_1 - x)^2 + y^2) + \frac{1}{m_2}(x^2 + y^2) \tag{1.42}$$

which simplifies to:

$$x^2 + y^2 - 2\frac{m_1 m_2}{m_1 + m_2} v_1 x = 0 \tag{1.43}$$

This can be written in the form:

$$(x - \frac{m_1 m_2}{m_1 + m_2} v_1)^2 + y^2 = (\frac{m_1 m_2}{m_1 + m_2} v_1)^2 \tag{1.44}$$

This equation has a geometrical meaning. It says that P lies on a circle that is described by:

$$R = x_{\text{center}} = \frac{m_1 m_2}{m_1 + m_2} v_1 \qquad \text{and} \qquad y_{\text{center}} = 0 \tag{1.45}$$

Lets look at elastic scattering of two particles with the same mass. Remembering

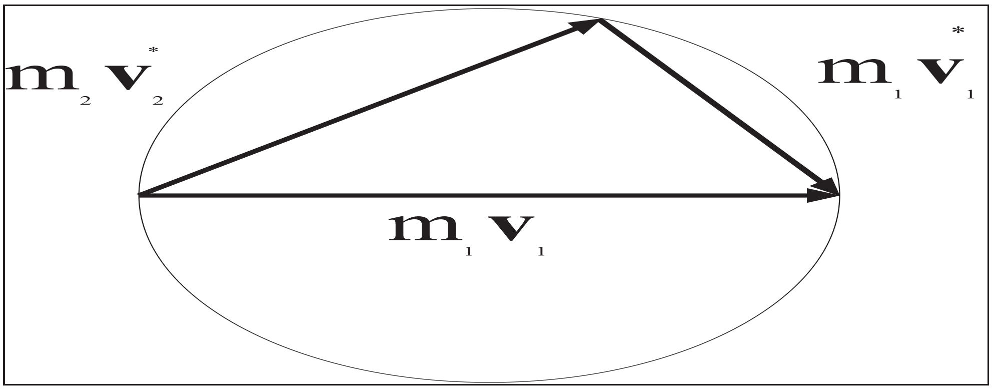


Figure 1.11: Scattering with $m_1 = m_2$

the theorem of Thales, we see that the angle between the two scattered particles is 90°. We also see, that the maximal transfer of momentum occurs for a head–on scattering. Then the incoming particle gives all it's momentum to the scattered particle. The transfer increases like $\cos(\alpha_{\text{scattering}})$. The highest scattering angle is 90° in the resting frame of a particle. If both particles collide with the same velocities but opposite direction, the over all maximum of $\alpha_{\text{scattering}} = 180°$ is possible. A particle can never change it's momentum more than all it's momentum in the rest

frame of the other particle.
The next case I would like to consider is that where $m_1 \ll m_2$. This is the case for a (fast) electron colliding with a (approximately resting) neutral atom (rest—gas). Taking the $\lim_{m1 \to 0}$ in equation (***) gives $R = x_{\text{center}} = m_1 v_1$. This is shown in figure (***)

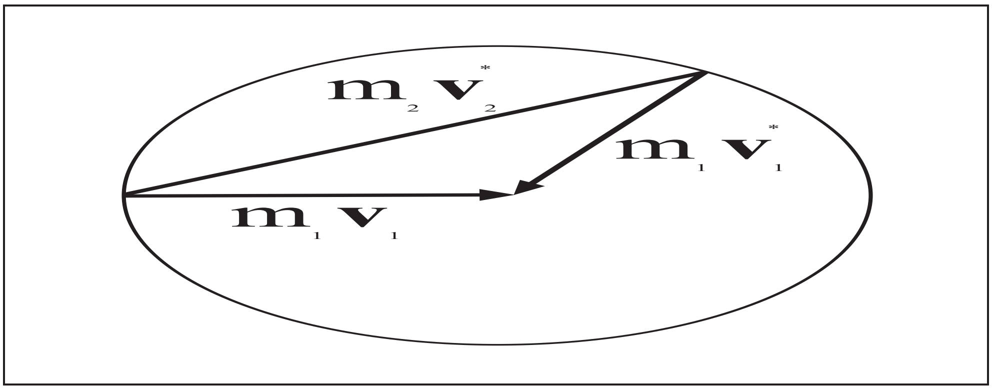


Figure 1.12: Scattering with $m_1 \ll m_2$

The momentum of the scattered electron can point in any direction, but will still have the same modulus. This means, that neutral atoms will in first approximation not contribute to the relaxation because they don't slow down the electrons sufficiently (as long as the pressure is low enough of course). However, since the electron can rapidly change it's direction when colliding with a neutral, these collisions will contribute to internal thermalization. They especially prevent a decoupling of the parallel and perpendicular energy distribution. This is one of the central assumptions I will always make.
The other case, where a fast neutral and a slow electron collide doesn't have to be considered. The reason for this is that the rest gas can be considered in thermal equilibrium with the surrounding (liquid helium) so that almost all the neutrals have a low velocity compared to the velocities of the light (and hotter) electrons.

1.5.2 The influence of scatterings on the guiding center theory

In this section I would like to show what collisions and scatterings do to the orbit of a particle. It will soon become clear, that depending on the collision–frequency, that is the number of collisions per second, these events can either be considered as a perturbating effect or as the main source of movement.
Figure *** gives a feeling for the effect of a scattering. If the scattering is a short

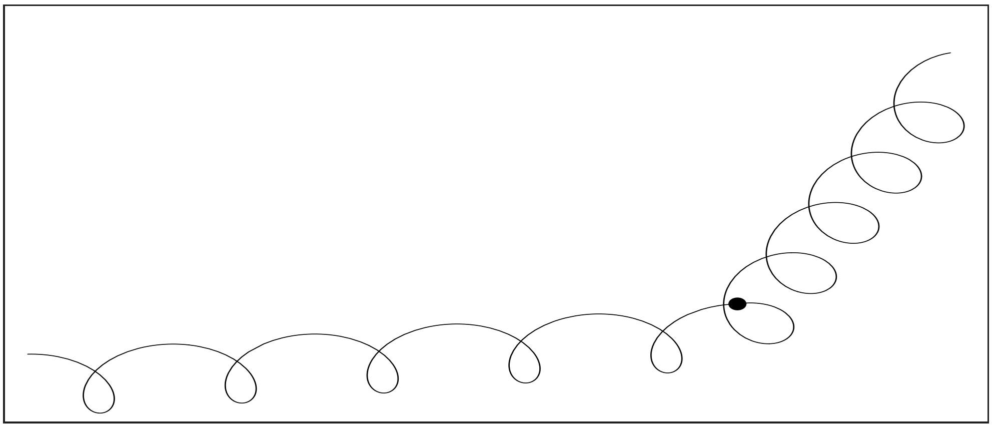


Figure 1.13: The guiding–center of a scattered particle seems to jump abruptly

event (which it normally is, because most of the orbit–changing happens during a very short time), we can consider it to be a binary event. So the momentum suddenly changes abruptly. After the scattering the particle will continue a guided gyromotion. So if you think of the scattering to be a short point–event then the effect it has macroscopically, is that it makes the guiding–center jump.

Collisions allow electrons to move across magnetic field lines

Electron–electron-scatterings happen at a rate that is roughly determined by:

$$\nu_{\text{scattering}} \equiv \frac{\overline{v}}{\lambda_D} \tag{1.46}$$

This gives a upper boundary. The length λ_D, the Debye-length is a length within which an electron always has a lot of neighbors. It's physical definition an meaning is given in (***).$\overline{v}$ is a typical (or average) velocity. The number of collisions is (about) the same for slow and fast particles because a slow particle is passed by lots of others and a fast one passes others – it's just a point of view. The electron–neutral-scattering frequency is determined by taking the formula for the pressure of an ideal gas. (See ***)

$$\nu_{\text{neutral}} \approx \frac{\overline{v_{e^-}}}{\lambda_{\text{gas}}} \tag{1.47}$$

The collisional frequency is normally small compared to the scattering frequency. Nevertheless collisions play an important role for radial transport processes.(See ***)

1.6 Holding mechanism for a pure electron plasma

The fact, that I am dealing with a pure electron plasma, that is an ensemble consisting of only electrons, gives me a nice advantage: I can in principle hold the plasma forever. In this section I would like to present, why this is possible. The whole argumentation is more or less a copy of [5]. The interesting consequences of this article make it worthwhile to present it again here.

I start with the equation for the total angular momentum of a set of particles in the MINERVA —trap.

$$P_{\angle} = \sum_{i=1}^{N} m\vec{v} \times \vec{r} + \int_V \frac{1}{4\pi c} \vec{r} \times (\vec{E} \times \vec{B})\, dV \tag{1.48}$$

The integration can be simplified for the electrode fields:

$$\begin{aligned} \int_V \frac{1}{4\pi c} \vec{r} \times (\vec{E} \times \vec{B})\, dV &= \frac{1}{4\pi c} \int_{r=0}^{\infty} \int_{z=-\infty}^{\infty} \int_{\theta=0}^{2\pi} r(-E_r \cdot B) dV \\ &= \frac{B}{2c} \int_r \int_z r^2 \frac{\partial \phi}{\partial r} dr dz \end{aligned} \tag{1.49}$$

As argumented in [5] this can be written as:

$$\frac{B}{2c} \int_r \int_z r^2 \frac{\partial \phi}{\partial r} dr dz = \frac{B}{2c} \sum_i q_i R_W^2 + \frac{B}{2c} \sum_i q_i r_i^2 \tag{1.50}$$

Here the simplification $z = \infty$ was applied and R_W stands for the radius of the electrodes. Using this much more descriptive formulation, the angular momentum equals:

$$\begin{aligned} P_{\angle} &= \underbrace{\sum_{i=1}^{N} m\vec{v} \times \vec{r}}_{mechanical} + \underbrace{\frac{B}{2c} \sum_i q_i R_W^2}_{constant} + \frac{B}{2c} \sum_i q_i r_i^2 \\ &= \sum_{i=1}^{N} m\vec{v} \times \vec{r} + \frac{B}{2c} \sum_i q_i R_W^2 - \frac{eB}{2c} \sum_i r_i^2 \end{aligned} \tag{1.51}$$

This formula shows what determines, how well a plasma can be held in a magnetic field. The mechanical part of the angular momentum is negligible, because the perpendicular velocities will cancel themselves out. If nobody applies any external torque, then the sum of the square radii stays constant. This only works for an ensemble that only consists of particles with one sign of charge. Neutral particles do not contribute to the radial term

A completely evacuated plasma trap, with no torque can thus store a pure electron plasma for ever. However, as long as the electrons can collide with neutral atoms, a movement across the magnetic field is still possible. This so called “neutral transport” is the dominating limit to life—times at poor vacua.
For sufficiently high vacua (P$\approx 10^{-9}$mBar) it shows that external torques due to misalignment of the trap to the field axis or field errors produce radial expansion.

Chapter 2

Statistical Treatment of plasmas

2.1 Introduction

Up till now all I have done is looked at the orbits of single particles in various field configurations. The thing to do now is solve the equations of motion for a large set ($\approx 10^7$) of particles, keeping in mind, that the electric field a particle sees depends on the positions of all the other particles and that the magnetic field depends on all their velocities.
This problem is much to complicated for a diploma student. That's why on normally tries to simplify life. Of course, we are for most purposes not interested in the exact description of every particle. It would be enough to know approximately, what the densities, energy distributions and other macroscopic variables equal to. In this chapter I will derive the results, needed for my thesis, that stem from a statistical treatment of the medium plasma. This formalism is very powerful.

2.1.1 Example for collective effects — Debye–shielding

The Debye–shielding is the easiest to understand effect, that arises from the fact that a plasma consists of a lot of particles. Putting it into a nutshell, it prevents electrical fields from entering the plasma. Lets start with an electron–plasma that has a Maxwell–Boltzmann-distribution.

$$f(\vec{x}, \vec{v}) = n_0 \exp(-\frac{mv^2}{2KT} + \frac{e\Phi}{KT}) \tag{2.1}$$

where Φ is the electrostatic potential of a particle in the plasma. This equation is easy to understand. Classically a particle can not be in a region without having the total energy of at least the potential energy at that point. That is why the second, potential-dependent term appears in equation (***). Integrating over the possible velocities (and normalizing to some charge density) gives the following expression for the number of electrons at a point with given potential Φ

$$n(x) = n_\infty \cdot \exp(\frac{e\Phi}{kT}) \tag{2.2}$$

This means, that at places where there is a blocking potential, the number of electrons is lower than at places with no potential. Off course we already see the direction this is taking: places with a high electron density act repelling to other electrons. So by the density distribution the influence of the blocking potential is counteracted.
This can be demonstrated quite nicely for a one-dimensional case with only electrons and the assumption, that the external potential is smaller than the thermal energy kT. The potential has to obey the self consistent poisson-equation:

$$\epsilon_0 \frac{d^2\Phi}{dx^2} = en_e \tag{2.3}$$

which under usage of (***) evolves to:

$$\begin{aligned} \epsilon_0 \frac{d^2\Phi}{dx^2} &= en_\infty \exp(\frac{e\Phi}{kT}) \\ &\approx en_\infty(\frac{e\Phi}{kT}) \end{aligned} \tag{2.4}$$

As boundary conditions I also choose something very simple. I ask for a given potential Φ_0 at $x = 0$ and $\Phi = 0$ everywhere else. This is about equivalent to a locally applied voltage. Of course this is the externally applied potential. The resulting potential is of the form:

$$\begin{aligned} \Phi(x) &= \Phi_0 \exp(-\frac{\mid x \mid}{\lambda_D}) \\ \lambda_D &\equiv \sqrt{(\frac{\epsilon_0 kT}{n_e^2})} \end{aligned} \tag{2.5}$$

2.2 The Maxwellian velocity distribution

One of the most important thing for the temperature measurement is knowing the energy distribution for a given situation. This is not a simple task, mainly because the system is not at thermal equilibrium. This basically means, that the rules of

thermodynamics can not be applied. Here I will show why the natural velocity distribution is Maxwellian.
Let's look at a medium, which is isotrop in at least 2 Dimensions. This can be a gas or the two dimensions of the MINERVA –plasma perpendicular to the magnetic field. For the probability, that one finds a particle in the velocity–space element $[v_1, v_1 + dv_1] \times [v_2, v_2 + dv_2] \times \cdots [v_n, v_n + dv_n]$ one can surely write

$$dP = f_1(v1) f_2(v2) \cdots f_n(v_n) dv_1 \cdots dv_n \tag{2.6}$$

Since the situation is per definition isotrop, the probability can only depend on the modulus of the velocity.

$$dP = F(v^2) dv_1 \cdots dv_n \tag{2.7}$$

Setting these two probabilities equal and differentiating *** v_1 gives the following result:

$$\frac{df}{dv_1} f_2 \cdots fn = \frac{dF}{dv^2}\frac{dv^2}{dv_1} = \frac{dF}{dv_2} 2v_1 \tag{2.8}$$

$$\Rightarrow \frac{1}{f_1}\frac{df_1}{dv_1}\frac{1}{2v_x} = \frac{dF}{dv^2}\frac{1}{F} \tag{2.9}$$

Since equation *** has to be valid for all velocities, we can equate both sides to a constant ($\equiv -s$). Solving the resulting differential equation for v^2 is easy.

$$F(v^2) = C_1 e^{-s \cdot v^2} \tag{2.10}$$

A negative value for s is excluded, since we of course want a converging probability. This derivation only requested isotropy and factorization into the dimensions. The second requirement is no problem for nonrelativistic particles, for which a velocity in one direction of course doesn't depend on the velocity in an other direction.
It is not easy to answer the question, whether the situation for a particle in a plasma is homogeneous. Intuitively you tend to deny this, because maybe, the particle sees an other particle in one direction, while there is none in an other one. The question can however be "answered" if you look at the path of a particle in the plasma.
The particle will fly around and scatter on other particles. If these scatterings are frequent and the particle is well inside the plasma, then it will encounter particles from a lot of different directions. So essentially for the single particle space will look isotrop, because of all "bumps" from the various sides. The only direction that could feel different, is the one parallel to the magnetic field. (Remember, that electric fields will not intrude into the plasma)

Collision parameters for an ideal gas

As a comparison to the MINERVA –plasma I will here just repeat some well–known (but maybe forgotten) formulas of statistical mechanics for gases. For an ideal gas the thermal velocity,mean free path and collision frequency are given by:[1]

$$\begin{aligned} \overline{v} &= \frac{2}{\pi}\sqrt{\frac{2kT}{m}} \\ \overline{\lambda} &= \frac{1}{\sqrt{2}\pi d^2}\frac{kT}{p} \\ \nu_{\text{collision}} &= \frac{\overline{v}}{\overline{\lambda}} = \frac{4d^2}{\sqrt{kTm}} \end{aligned} \tag{2.11}$$

where d stands for the diameter of the gas particles.

	T=300K			T=4K		
Gas	$\overline{v}$	$\overline{\lambda}$	ν_c	$\overline{v}$	$\overline{\lambda}$	ν_c
He	$708\frac{m}{s}$	$60nm$	12GHz	$82\frac{m}{s}$	776m	0.1Hz
H_2	$1418\frac{m}{s}$	$400nm$	3.5 GHz	$164\frac{m}{s}$	4.8km	0.03Hz

Table 2.1: Typical collision parameters for the two rest gases in MINERVA . The values for the normal temperature are taken for 1000mBar, the ones for LHe–temperature at $1 \cdot 10^{-9}$mBar

This shows clearly, that the movement of a particle in a “normal” gas is totally determined by collisions with other particles. Actually the values of the mean free path and thus collision frequency for the cold gas are quiet doubtful because of course the main contribution here would be collisions with the walls of the vacuum–chamber.

2.2.1 Local and global thermal equilibrium

The problem for the Maxwellian velocity distribution is, that you rely on a situation, that doesn’t change on the time–scale of a lot of collisions. The simplest example hereof is thermal equilibrium. But because I am studying a situation, where a system is relaxing to a temperature, I can of course not use this assumption. Instead I will consider the system to be in local thermal equilibrium. This means, that thermalization has started, but is not completed over the whole plasma. So locally I can already speak of a temperature (and thus use velocity distributions), while this might not make any sense for the whole ensemble.

[1] See [4] on pages 314–318

2.3 The plasma fluid equations

Introduction

To be able to estimate the time it takes the plasma to thermalize, that is to relax to a global temperature the ordinary thermo—dynamics or statistical mechanics are not suitable. The reason herefor is, that one there normally relies on a equilibrium. Here however, I am looking at a situation, that is per definition not in thermal equilibrium. So I have to look for a set of equations, that describe transport processes.

2.3.1 Derivation of the fluid equations

In this chapter I will try to show how to deduce the plasma fluid–equations. In the following I always assume, that all requirements for a statistical treatment are fulfilled. Without caring to much about mathematical details I introduce the space–velocity distribution $f(\mathbf{x}, \mathbf{v}, t)$ that gives the number of particles within $[\Gamma, \Gamma + \Delta\Gamma]$

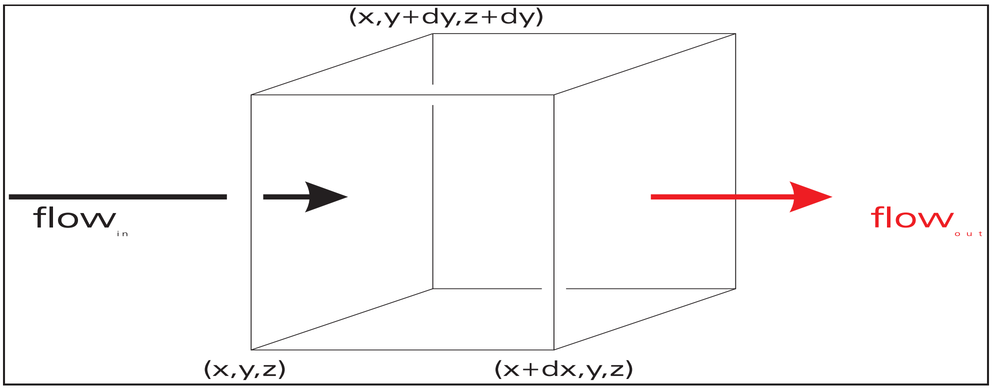


Figure 2.1: The difference in particle density is given by the difference in flow into and out of a volume element

Because the other two directions are analog, I will just look at the x–direction. For the two flows I can write:

$$\begin{aligned} F_{\text{in}} &= f v_x dydzdt \mid_x \\ F_{\text{out}} &= f v_x dydzdt \mid_{x+dx} \end{aligned} \tag{2.12}$$

so for the change in the number of particles I get:

$$\frac{F_{\text{out}} - F_{\text{in}}}{dxdydzdt} = \frac{(v_x f)|_{x+dx} - (v_x f)|_x}{dx} = \frac{\partial(v_x f)}{\partial x} \tag{2.13}$$

So for the spatial term I get:

$$\frac{\partial(v_x f)}{\partial x} + \frac{\partial(v_y f)}{\partial y} + \frac{\partial(v_z f)}{\partial z} = -\frac{\partial f}{\partial t} \tag{2.14}$$

The negative sign is there, because a positive derivation indicates, that particles have flown out of the element. Until now I have only included the fact, that particles can stream through space. Of course they can also enter the phase–space element, by changing their velocity. Considering this equation (***) expands to:

$$\begin{aligned} \frac{\partial(v_i f)}{\partial x_i} + \frac{\partial(a_i f)}{\partial v_i} &= -\frac{\partial f}{\partial t} \\ \Rightarrow \qquad \frac{\partial f}{\partial t} + v_i \frac{\partial(f)}{\partial x_i} + \frac{F_i}{m}\frac{\partial(f)}{\partial v_i} &= 0 \end{aligned} \tag{2.15}$$

The last equation assumes, that a_i does not explicitly depend on v_i which is always the case for the Lorentz–force. It is called the "collisionless" Boltzmann–equation.
The problem is finding the right expression for the force F_i. Of course it would not help anything if one has a statistical description but still a force–field that depends on the single particles. That's why one normally writes:

$$\frac{F_i}{m} = \frac{e}{m}[\overline{\mathbf{E}_i} + (\mathbf{v} \times \overline{\mathbf{B}})_i] \tag{2.16}$$

Here the two fields $\overline{\mathbf{E}_i}$ and $\overline{\mathbf{B}}$ specify average fields. These averaged fields just depend on the position in the plasma and due not take the exact locations and velocities of all the particles in account. This is the reason why one speaks of a "collisionless" equation. A collision is a event where the field a particle is seeing is mainly determined by the position of an other one. The average fields contain external fields (electrodes etc.) as well as "internal" ones stemming from the geometry of the charge distribution. Explicitly inserting the Lorentz–force for the force–term gives us:

$$\frac{\partial f}{\partial t} + v_i \frac{\partial f}{\partial x_i} + \frac{e}{m}(\vec{E} + \vec{v} \times \vec{B})_i \frac{\partial f}{\partial v_i} = 0 \tag{2.17}$$

Which is called the Vlasov–equation. (Sometimes together with the Maxwell–equations for the charge distribution.
The "collisionless" Boltzmann–equation can be expanded in order stay abreast of collisions. The normal approach here is to just insert a new term:

$$\frac{\partial f}{\partial t} + v_i \frac{\partial f}{\partial x_i} + \frac{e}{m}(\vec{E} + \vec{v} \times \vec{B})_i \frac{\partial f}{\partial v_i} = \frac{\partial f}{\partial t}_{\text{collision}} \tag{2.18}$$

2.3.2 Collision – terms

Of course it is not easy at all to find a suitable and plausible collision–term. I will just present two possible ones, the Krook–term as the "easiest" one and the Fokker-Planck–term as the most common one.

The Krook–term

$$\frac{\partial f}{\partial t}_c \equiv \frac{f - f\text{max}}{\tau} \tag{2.19}$$

$f\text{max}$ is the value for the Maxwellian distribution and τ specifies the time scaling. Krook chose $\tau \equiv \nu^{-1}_{\text{collision}}$ that is the number of collisions per second.
The Krook–term has some easy to understand properties:

- A system always tends towards a Maxwellian distribution
- It is linear in the number of collisions per seconds. This means it vanishes for a non–collisional system and grows dominant if a system has a lot of collisions (e.g. An ideal gas).

Basically the term is quiet intuitively right. The main disadvantage is, that it contains no real physics.

The Fokker-Planck–term

The Fokker-Planck–term comes from a different approach. If you look at what a collision (see ***) does to a particle, you are reminded of the Brownian-motion. Thus like the people studying that motion one introduces:

$$\Psi(\mathbf{v}, \mathbf{\Delta v}) = P(v_{\text{initial}} = \mathbf{v} \text{ changes by } \Delta v \text{ within } \Delta t) \tag{2.20}$$

Of course this probability always has to be understood as a statistical size. Using this function the time–development can be written as:

$$f(\underline{x}, \underline{v}, t) = \int f(\underline{x}, \underline{v} - \Delta v, t)\Psi(\underline{v} - \Delta v, \Delta v)d^3(\Delta v) \tag{2.21}$$

which can be written as:

$$f(\underline{x}, \underline{v}, t) = f(\underline{x}, \underline{v}, t - \Delta t) - \frac{\partial}{\partial \underline{v}}(f\underline{\langle \Delta v \rangle}) + \frac{1}{2}\frac{\partial^2}{\partial \underline{v} \partial \underline{v}}(f\underline{\langle \Delta v \Delta v \rangle}) \tag{2.22}$$

$$\underline{\langle \Delta v \rangle} \equiv \int \Psi \underline{\Delta v} d^3 \Delta v$$

$$\underline{\langle \Delta v \Delta v \rangle} \equiv \int \Psi \underline{\Delta v \Delta v} d^3 \Delta v$$

This approach brings the problem of finding Ψ. This is not easy at all. Normally one calculates it by taking an average amount of collisions and using the Rutherford–cross–section for the probability of a velocity change. I will not go further into details.

2.4 Thermalization speed

In this section I will ask how long it will take the plasma to achieve a uniform (over the size of the plasma) temperature. I will show, that the internal thermalization is faster than the cooling to the temperature of the surrounding. What this means is that we can really speak of a medium with a temporarily "defined" temperature that is cooling down.

2.5 Plasma heating

2.6 Excited states — The dicotron oscillation

Up till now the electron plasma has always been treated as a "static" object. Two effects caused movements and density differences. The first is a temperature gradient and the second negligible microfields which are not shielded by Debye-screening. The state to which the ensemble tends was always a static one.
This is however not necessary. The classical approach to plasma dynamics (see for instance [2]) is to start with the fluid and the Maxwell equations. By introducing perturbating fields or explicitly inserting wave-solutions, one sees that a set of wave-modes can propagate through a plasma. The varying quantity is the electron density. The set of plasma equations gives a dispersion relation. It is only calculatable exactly for several simple cases. These waves are classified as usual by giving three wave-vector components. Any "hot" plasma is bound to have some dynamics. Since it is not possible to detect axial waves, I will only discuss radial ones. Plasma wave which consist only of radial components are called Dicotron waves. As shown in (***) systems with Dicotron waves have lowered lifetimes. This is why one also speaks of the Dicotron-instability.
Radial oscillations can be detected and damped. The principle of this is shown in figure (***). A radial oscillations lets the center of charge move. If you take an electrode (S1&S2) then the opposite sides will be charged depending on the position of the plasma center of charge. This charging will happen at the frequency of the oscillation. Looking at the Signal on a segmented electrode with a spectrum analyzer should reveal any oscillation.

Damping an oscillation is easy if one understands the detection principle. All you have to do is take the induced signal on one segment and feed it back (amplified) to the other segment.

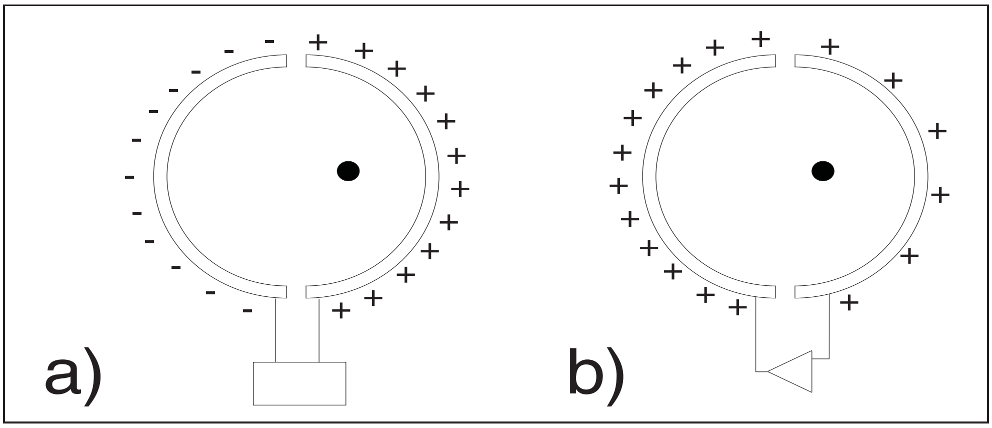


Figure 2.2: Both detection (a) and damping (b)of radial oscillations are done by a segmented electrode.

2.7 Calculation of relaxation rate

In this section I will calculate the cooling rate of an electron plasma considering only synchrotron radiation. At first I make the assumption, that the cooling rate is longer than the internal thermalization speed. This makes sense considering the times calculated in (***) and (***).
Assuming that the radial and the axial temperatures $T_\perp$ and $T_\parallel$ are about equal, there is no temporal change in temperature ($\frac{\partial T}{\partial t} \equiv 0$) and the total energy is preserved I can write:

$$\frac{dT_\parallel}{dt} = -2\nu_1(T_\parallel - T_\perp) \tag{2.23}$$

$$\frac{dT_\perp}{dt} = -\nu_2(T_\perp - T_\parallel) \tag{2.24}$$

$$\nu_1 = \nu_2 \tag{2.25}$$

The factor two in the first equation stems from the fact, that there are two perpendicular degrees of freedom (which carry twice as much energy) trying to force the parallel component into thermal equilibrium. The last one is a consequence of the conservation of energy.
If this thesis is to make some sense (consider the title) then there will be a time dependent change in temperature. Including this expands the equations for the time-evolution to:

$$\frac{dT_\parallel}{dt} = -2\nu(T_\parallel - T_\perp) + 2\frac{\partial T_\parallel}{\partial t} \tag{2.26}$$

$$\frac{dT_\perp}{dt} = \nu(T_\parallel - T_\perp) + \frac{\partial T_\perp}{\partial t} \tag{2.27}$$

For the temporal change of the temperature I can now choose a term. As discussed in (***) the main contribution will be the synchrotron radiation emitted. This radiation loss is inserted as the change of the perpendicular temperature. The parallel temperature will in first approximation not change via radiation losses. This gives me:

$$\begin{aligned}\frac{dT_\parallel}{dt} &= -2\nu(T_\parallel - T_\perp)\\ \frac{dT_\perp}{dt} &= \nu(T_\parallel - T_\perp) - \frac{4}{3}\frac{e^4B^2}{m^3c^5}T_\perp \end{aligned} \tag{2.28}$$

Internal thermalization times are faster than the radiation cooling. I can thus assume that $T_\parallel = T_\perp \equiv \frac{1}{3}T$.

$$\frac{dT}{dt} = \frac{dT_\parallel}{dt} + 2\cdot\frac{dT_\perp}{dt} = -\frac{8}{9}\frac{e^4B^2}{m^3c^5}T \equiv -\frac{1}{\tau}T \tag{2.29}$$

The time-evolution in a synchrotron-dominated plasma will be:

$$T(t) = T_0 \cdot \exp\left(-\frac{t}{\tau}\right) \tag{2.30}$$

This solution is as already stated in (***) unphysical since the temperature would approach 0K. It is however clear that the plasma can not cool down (much) more than it's environment. Therefor at low temperatures the formula for the temperature-evolution will fail. The reason is that I have not included any thermal radiation corrections (imagine both plasma and environment to be black bodies). I will also renounce to do this because these effects only get noticeable for temperatures in the meV range. Such low energies can not be measured with my apparatus due to the measuring method (see ***). Also the MINERVA plasma will not cool down that much anyway.

Chapter 3

Experimental setup

3.0.1 Introduction

In this chapter I will present the setup of the MINERVA -experiment. Basically all the parts of MINERVA fall into one of the following categories:

Plasma environment In this category you find all the machines etc. that are needed to provide an environment, in which a plasma can be produced

Plasma creation and handling All the devices needed to produce, store, dump, move and measure an electron plasma

Control / Read-out–system The operation of all the machines as well as the data-acquisition system

3.1 Plasma environment

There are two essential environmental demands to fulfill. The first is a axial magnetic field for a radial confinement. In my case this is produced with a superconducting coil. The second requirement is a low pressure. As shown in (***) neutrals cause radial transport which is not wanted. Even more important is that I want to measure the properties of a plasma and not those of a (ionized) gas.

3.1.1 The magnet

The magnet I use for MINERVA is a 2 Tesla super-conducting magnet.

Figure 3.1: The super-conducting coil of the MINERVA system

length	1m
maximal field	2T
length of 99%-field	≡ 80cm
operating temperature	4K

Table 3.1: Characteristic values of the MINERVA magnet.

The magnet is an “ordinary” super-conducting coil. For further information I would advise to read in *******.

3.1.2 Vacuum

There are three issues of vacuum-technology involved in MINERVA . The first is creating a vacuum. I have pressure requirements of about $5 \cdot 10^{-9}$mBar. The advantage of the super-conducting magnet is that I operate a big part of the system at 4K. This means that the walls of the vacuum vessel will act as a big cryo-pump and freeze out anything but hydrogen and helium (and other noble gases). So I don't even have to pump down to the desired pressure. In order to not have all the surfaces coated with ice and other frozen junk I pump the system to about $4 \cdot 10^{-7}$mBar before cooling down.
The second task is maintaining a low pressure. Just because large parts of MINERVA are immersed into liquid helium does not mean the vacuum would not decrease over time. All the parts between the cold part and room temperature will continue to outgas gases stored in the material. This outgassing can not be taken by the cryo-pump over a longer time. And of course in case of a small vacuum leak ($< 10^{-8}$mBarls^{-1}) it is nice to be able to pump on a system to maintain some pressure.
The last requirement is safety. Because of the cold it would be a disaster if the system would out of some reason vent to air. The instreaming air and especially the water vapor would freeze out and form blocks of ice in the system.

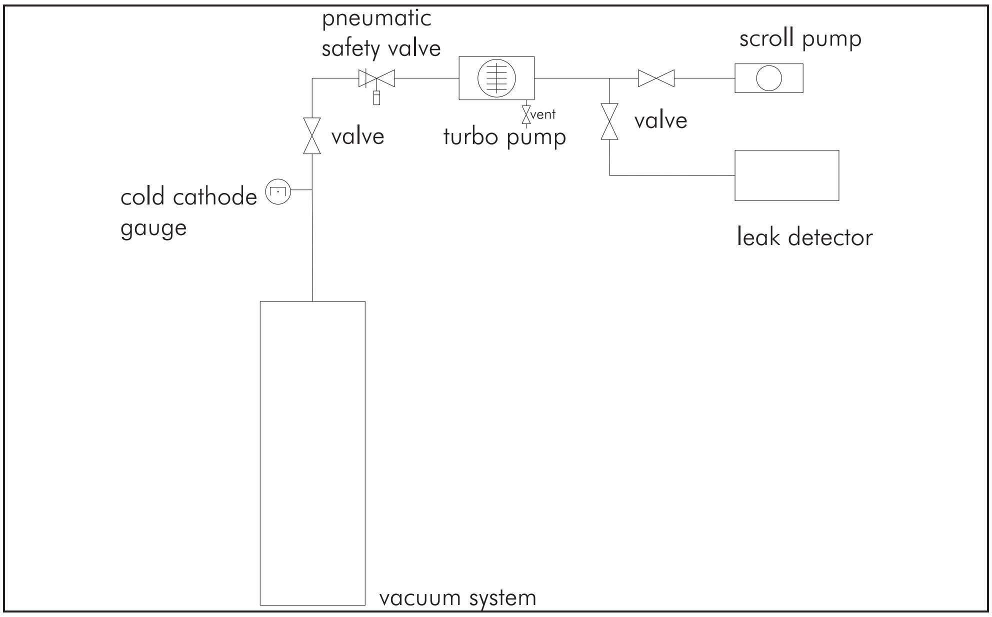


Figure 3.2: Schematic overview of the vacuum system. MINERVA is a two stage vacuum setup

The pumping is done by two pumps. First a fore-pump that achieves a pressure of about 10^{-2}mBar. This is done by the mechanical, oil-free scroll pump(60 ls^{-1}) at typical pressures for N_2. At about 10 mBar the turbo is started. The turbo is backed by the scroll pump and has a final pressure (without gas-load and powerful backing-pump) of $1 \cdot 10^{-10}$mBar. It's pumping speed is about $60ls^{-1}$ in the pressure range 10^{-3}–10^{-9} mBar for N_2 and about 40% less for helium.
This system has several advantages:

- It's nearly **oil-free**. Especially rotary prepumps are dangerous in case of a pump failure, because oil is sucked into the vacuum system and pollutes the clean surfaces.
- It's **simple**. Unlike other systems no pumps have to be closed of during some phase of pumping. In addition no baking or other treatment is needed.

but also some drawbacks:

- It's **open to air**. In order to prevent air streaming in, in case of a pump failure an additional valve is needed.
- It is probably **not operational at full field**. Turbos do not like to run in high magnetic fields, because they heat up due to induced currents. Note that the turbo is spinning at 1500Hz.

The pumping speeds of the two pumps are not reached in practice, because the flow is limited by the length (and diameter) of the tube going to the cold part. I would estimate the pumping speed to be in the range of a few liters per second.
If need be a helium leak detector can be installed behind the turbo-pump while the system is running. A leak detector is basically a mass-spectrometer that measures the flow of a specific gas. (in $\mathrm{mBar\,l\,s^{-1}}$). In case of a power cut the automatic valve shuts and seals of the system. This is not a 100% protection since it fails if the pumping system stops out of some other reason (overheating of turbo etc.) but covers the most important source of problems.
The pumping speed of the cold surface is not known precisely. It is especially high in the beginning and decreases as the surface is fully coated with a thin layer. Anyway it will not pump faster than the turbo-system, since again conductance of the tube is the limiting factor of the "cryo-pump".

3.2 Plasma handling

Plasma handling in MINERVA consists of three parts. First of all I have to "produce" a plasma. The main goals here are to get a lot of electrons in a small amount of time, not get any ions into the system and not heat the system up a lot. Then I have to be able to store and modulate the plasma. The radial storage was already described in ***. And finally I would like to analyze the plasma, so some kind of detector is needed.

3.2.1 Production

Free electrons are produced in a simple way. All you have to do is heat a material to a temperature, at which a lot of electrons have an energy larger than the work function of the material. This system although simple has the disadvantage that explicitly don't want the system to be heated up. So in order to reduce this problem the aim is to take a material that has a low work function and a lot of electrons. A good choice is Barium-oxide.

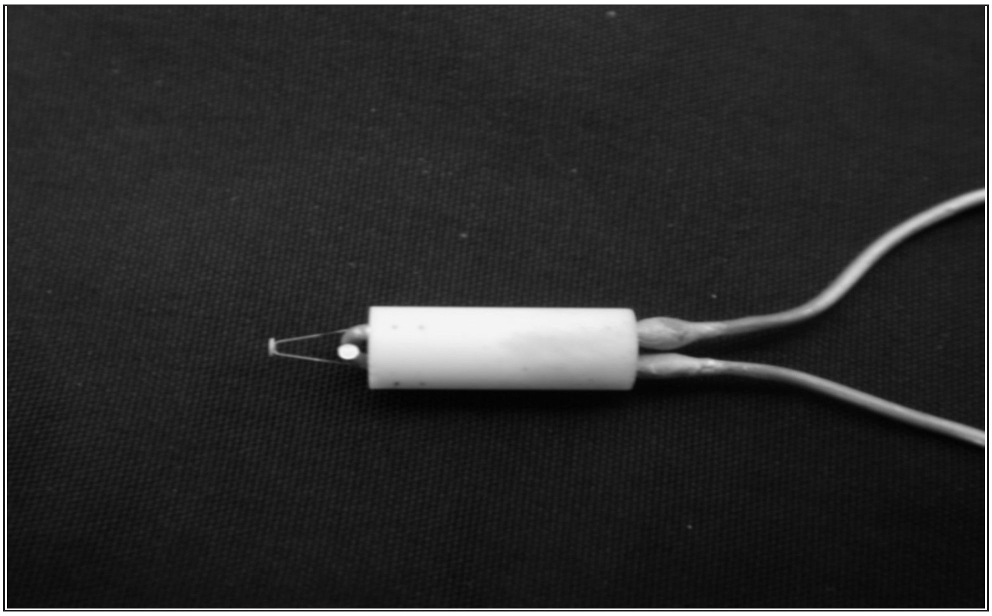


Figure 3.3: The electron filament used in Minerva. The electrons come from the small disc in the front

The typical operational temperature of the Barium-oxide is 1000–1100K at which around 100μA of current can be drawn out of the filament and 1.5W of heat are dissipated.

3.2.2 The plasma trap – axial confinement

The trap used in MINERVA is a cylindrical Penning-trap. The idea behind the Penning-trap is to create a confining potential in axial direction. This potential should be confining for all radii. This is basically all I need for an electron plasma. For this task cylinders are set to a given (negative) voltage. This potential has been calculated in ***.

There are five electrodes. The middle one is cut into two half-moon sectors. It can

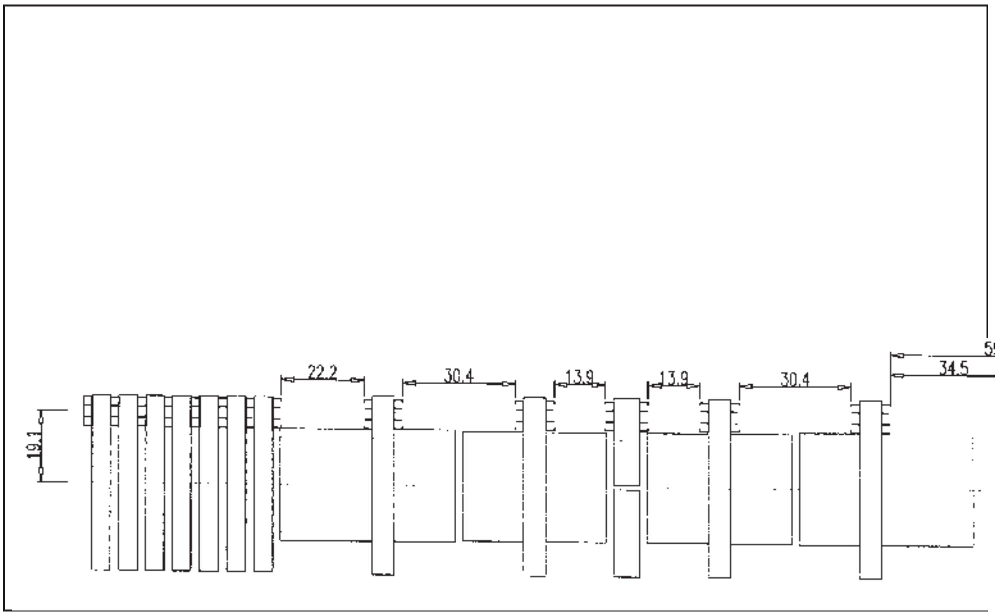


Figure 3.4: Technical drawing of the MINERVA trap

radius of the Wall	1km
length of outermost electrodes	1m
length of inner electrode	2T
overall length of the trap	≡ 80cm

Table 3.2: Dimensions of the MINERVA -trap

thus say something about radial displacements or be used to cause such. On both sides of that one there are two cylindrical electrodes. If I set two of the electrodes on

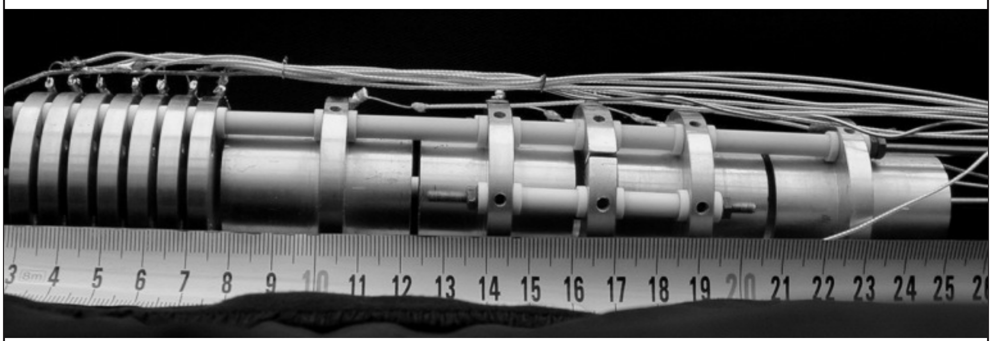


Figure 3.5: The MINERVA -trap is made of gold-plated copper

a potential, then electrons can be stored between them.
The trap is held together by threaded rods to which the trap is screwed. The typical tolerances of the trap itself are in the region of 0.1mm and 0.5°. That means that the electrodes are not tilted more than 0.5° with respect to the central axis. The dimensions of the electrodes can be trusted while the distances between electrodes have to be adjusted carefully.

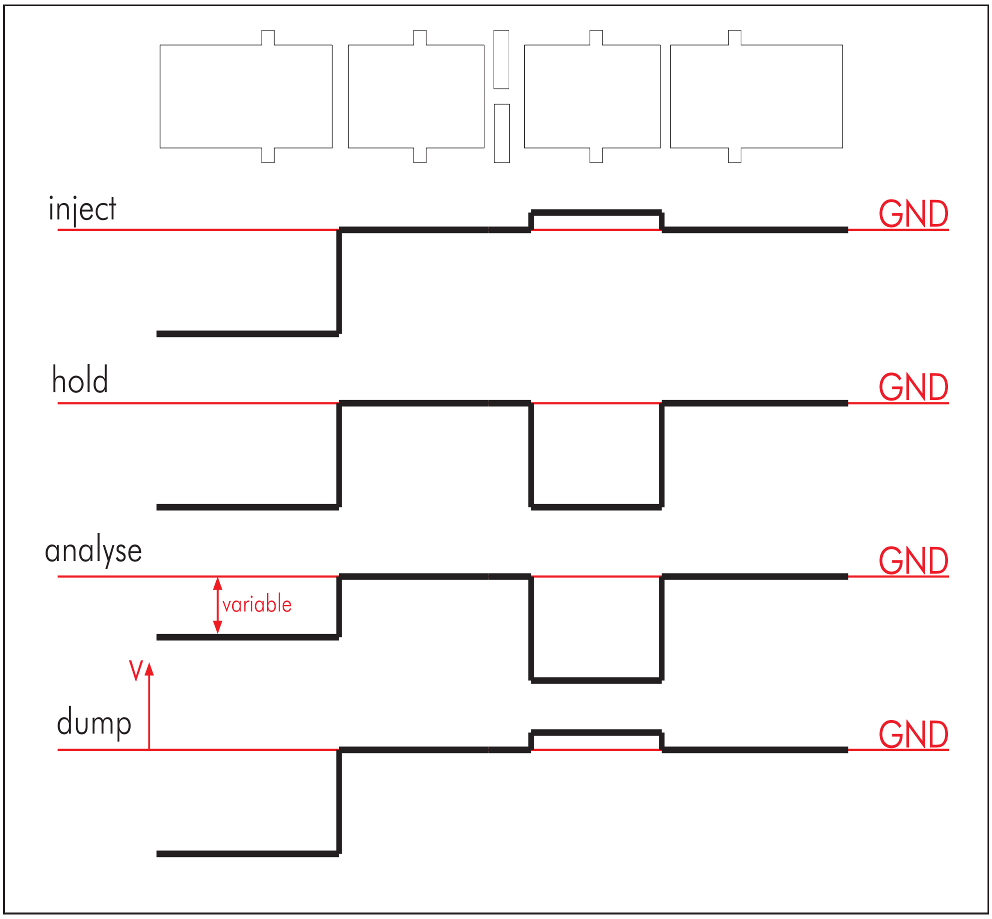


Figure 3.6: Production, hold, analyze and dump cycle on the electrodes

Figure (***) shows how the potentials are set during a plasma study. The reason why I don't use the lowest electrode (L1) is that the electrical pulse would be seen to much on the detectors.(Capacitive coupling)

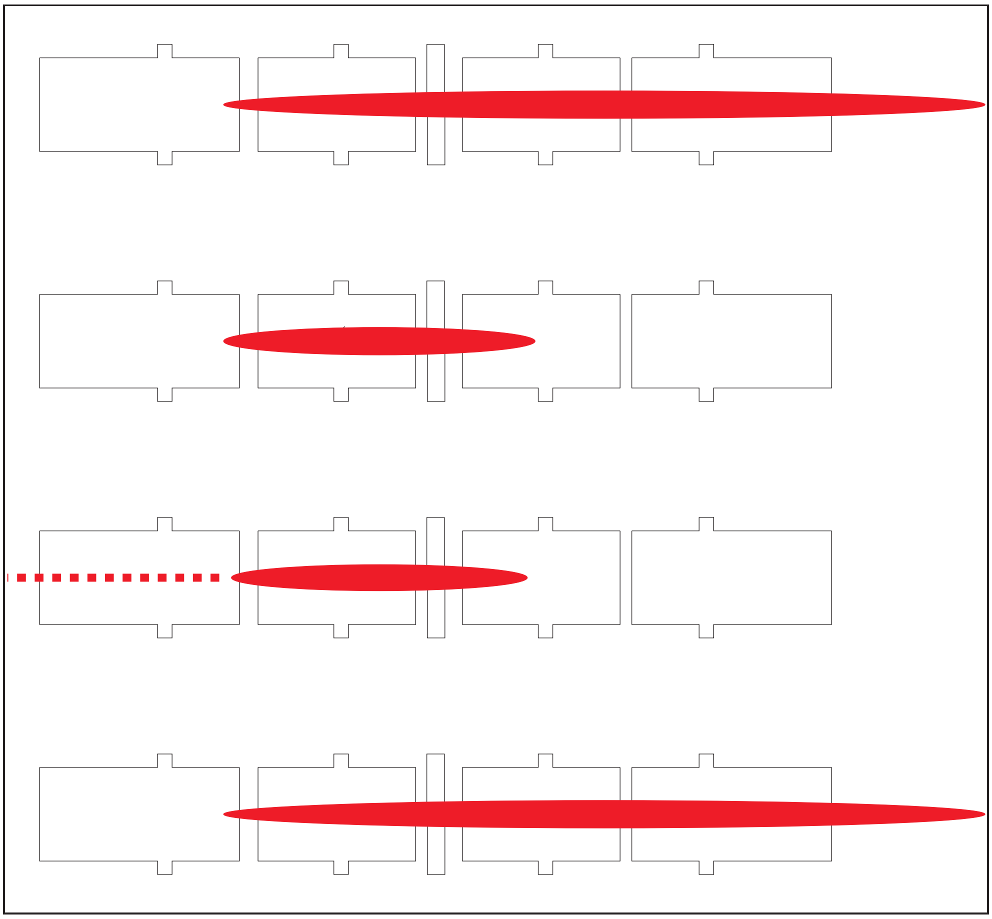


Figure 3.7: Location of the electrons during a cycle

In this figure you see where the electrons are during the analyzing cycle.

Production The “entrance”-electrode (L4) is open, the “exit”–L1 shut. The electrons fill the trap

Hold Both electrodes are set to a negative voltage. The electrons are contained in the trap and do the doo woop.

Analysis The “exit”-electrode is set to some voltage. Some electrons escape

Dump The “exit” is set to ground potential. The remaining electrons escape. The trap is ready for the next plasma

3.2.3 Charge detection — The Faraday cups

The last element of the plasma handling is measuring the plasma parameters. In my case (see ***) I want to measure numbers of electrons as a function of radius and energy. It is very simple to build detectors for charge. In the case of MINERVA they are just discs attached to wires. If charge is dumped it flows off and is measured. In order to get a radial resolution there are seven discs. They have increasingly big holes in the middle. So they first electrode (P7) only collects the charge between R_7 and R_{wall}. The next plate sees the electrons between R_6 and R_7 (off course it is blind to $[R_7, R_{\text{wall}}]$ because it is behind P7). This continues until the final plate P1 has no hole in it, that means it collects the central particles.

Figure 3.8: The MINERVA charge detectors are rather simple – They are just a series of plates

R_1	0.5mm	R_2	1mm
R_3	1.5mm	R_4	2mm
R_5	2.5mm	R_6	3mm
R_7	3.5mm	R_{wall}	0.5mm

Table 3.3: Radii of the "Faraday cups"

3.3 Control / Read-out system

The whole MINERVA –experiment has to be controlled and data needs to be taken and stored somewhere. In this section I will present how this is done. I will not emphasize the control of the magnet since it is like the rest of the magnet just a "off-the-shelf" box.

3.3.1 Requirements

- Set all electrodes to potentials. For storage the voltages will be in the range of about -100V—0V. Maybe during the dumping process one would like a slightly positive voltage.
- For at least one (better two) electrodes (namely the exit electrode and preferably also the entrance electrode) I need to apply pulses. These pulses are typically in the time scale of milliseconds. The pulses are made out of ramps and constant plains and do not have to be free waveforms.
- The pulses have to be applied in a given temporal sequence. Especially the time between the inject and dump pulse must be adjustable. *** suggests an accuracy of around 1ms.
- One must be able to control the heating current and acceleration voltage for the electron filament in order to get the appropriate amount of electrons into the trap
- The Faraday cups should be set to a known (positive) potential.
- The magnetic field has to be set to a given value
- The charges dumped on the Faraday cups must be measured. This requirement can be changed if you assume a constant temperature. Then all you need is some information on the size of the plasma and the total charge. This should make life simpler.
- I need to know the pressure. This knowledge doesn't go directly into any equation but if the pressure is to high *** the plasma will expand to fast. Expansion of a pure electron plasma heats the plasma because the electrostatic energy goes into kinetic energy of the particles. Extra heating would of course falsify the results.
- It is important to detect and damp any plasma instabilities.
- The system should be computer controllable, making it possible to take a lot of data.

3.3.2 The MINERVA – solution

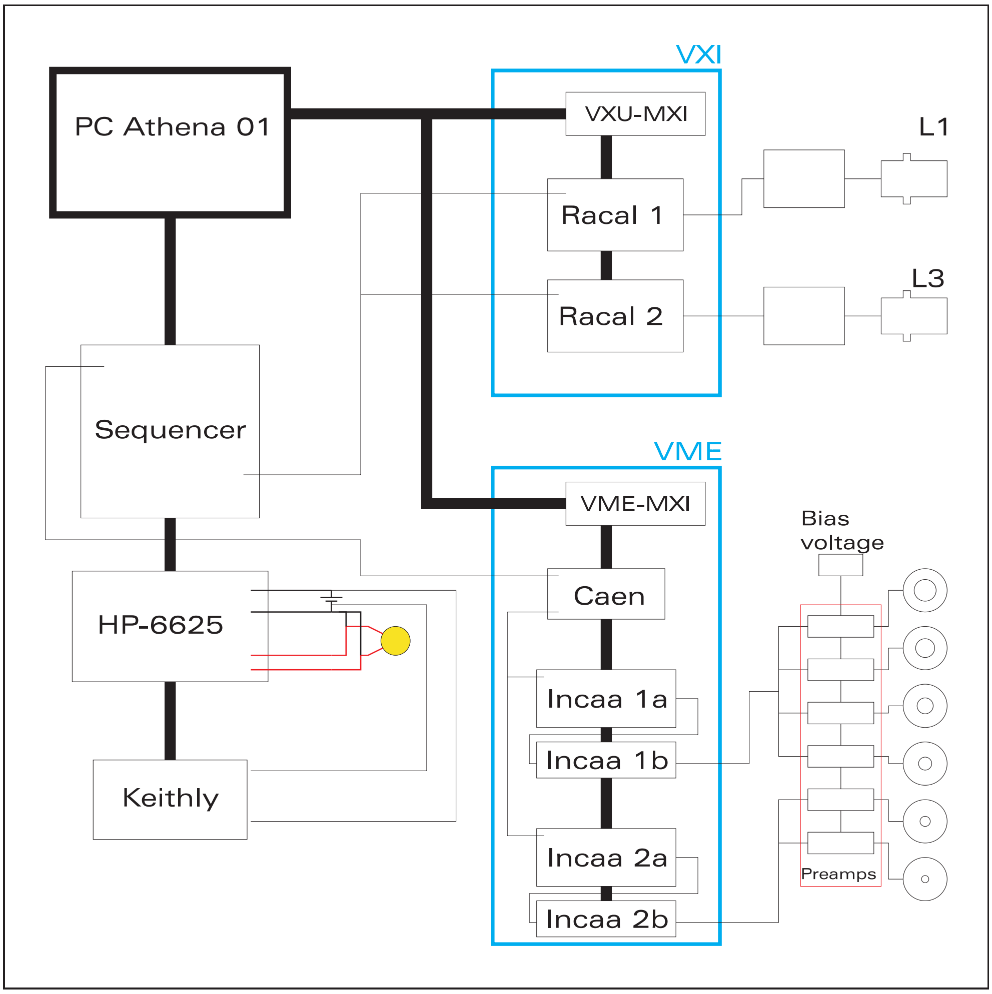


Figure 3.9: Schematic overview of the MINERVA control system.

Figure *** shows the control system of MINERVA . It is organized from the point of view of the different buses involved in the electronic system.

Nr.	Name	Description
1	PCAthena01	Windows NT computer running Labview
2	VXI-MXI-2	Controller that converts the two buses VXI and MXI
3	Racal1	Programmable waveform generator
4	Racal2	see Racal 1
5	Pulser 1	Basically just an amplifier that boosts the low output voltage of the Racal to the designated level. In addition it allows you to set a constant offset.
6	Pulser 2	see Pulser 1
7	VME-MXI-2	Another inter-bus controller, this time to the VME-bus
8	Incaa 1a	Analog-Digital-converter. The Incaa ADC reads 4 signals coming from the detector.
9	Incaa 1b	In front of the ADC is an amplifier, that converts the incoming signals to a suitable size.
10	Incaa 2a	As the Incaa 1a. For the rest of the detectors.
11	Incaa 2b	see Incaa 1b
12	PA1–PA7	Charge sensitve preamps sitting directly outside the system
13	Caen SPA	The "start-button". All this device does is set the ADC's and the sequencer into action.
13	HP 6625a	This powersupply controls the heating current and acceleration energy of the filament.
14	Keithly	A multimeter. Has several usages, mostly needed, to measure the electron current.
15	Sequencer	The guy with the stopwatch. The sequencer controls the time between the inject and the dump pulse. It has an accuracy in the μs region and a time range of seconds.

Table 3.4: The components of the MINERVA control system

Figure 3.10: The whole MINERVA experiment (with exception of the magnet) is controlled by one Labview program.

Chapter 4

The temperature measurement

4.0.3 Introduction

In the last 3 chapters I have introduced everything required to understand the MINERVA -plasma. I have calculated the time-evolution predicted by a synchrotron radiating plasma, presented the tools used in MINERVA and showed the techniques that allow me to handle a plasma.
This chapter will illustrate how I actually got my temperature evolution, starting from the raw data.

4.1 Measuring the temperature

What is a temperature? Before measuring it, it is worthwhile thinking about this for a moment. I have shown that for the situation of MINERVA the velocity distribution is Maxwellian (That is equivalent to a Boltzmann-distribution of energy). The Boltzmann-distribution has the form:

$$\begin{aligned} P(E) &= A \cdot \exp{(-\frac{E}{kT})} \\ A^{-1} &= \int_0^\infty P(E)dE = kT \\ \langle E \rangle &= A \int_0^\infty EP(E)dE = \frac{3}{2}kT \end{aligned} \tag{4.1}$$

The expression kT has two meanings:

- It is proportional to the energy expectation value. This is what we generally connect with the word temperature.
- It acts as a scaling factor for the width (e-folding–width) of the energy distributions. Hot media have a broader energy distribution than cold ones.

In MINERVA I will use exactly the second characteristic to measure the temperature. I will measure a part of the Boltzmann-distribution and extract the e-folding constant.

4.1.1 Calculation of the expected signals

Potentials along the axes

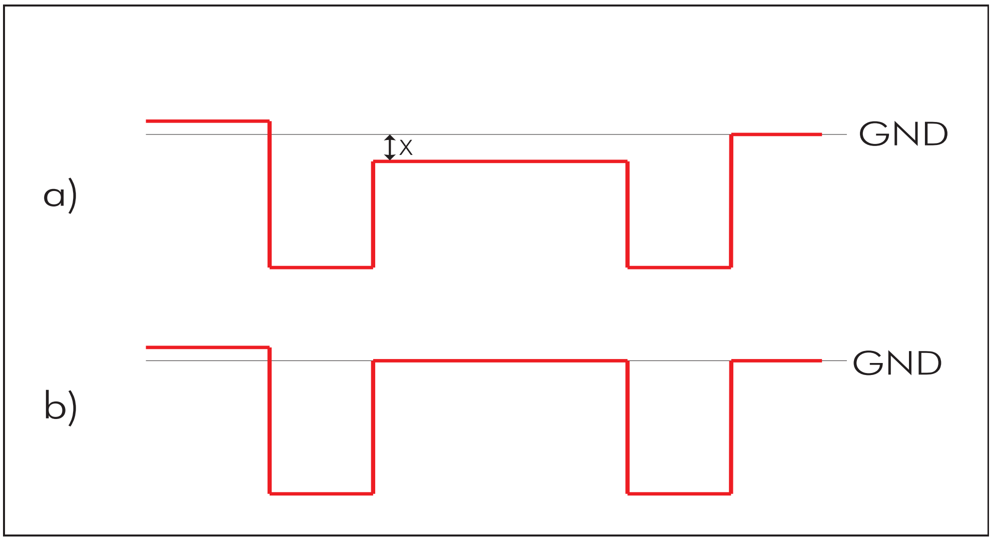


Figure 4.1: The potential along the central axis with (situation a) and without (situation b) a captured plasma

A—The Faraday cups The Faraday cups are set to the lowest potential. If they are allowed to go there, the electrons will preferably go to these collectors

B—The electrodes Since I want to capture electrons between the two electrodes, I will apply a high confining potential to the electrodes. This barrier prevents electrons from escaping or entering into the trap region

C—The trap region. Due to Debye-screening, there are no internal potentials. This is a region of a constant potential. Off course this potential is positive compared to the "outside–world", since it would take work to move an additional electron into the plasma (forgetting about the electrodes for a second). This potential energy is determined by the Poisson-equation which connects charge densities and potential. In a static case it will be constant on the "surface" of the plasma.

D—The world Production place of the electrons, here the electrons can move quite unopposed and especially flow of to ground. I consequentially call this potential GROUND

Let's start with the assumption that the confining potentials ($V*$) are infinite. Then the energy distribution within the plasma will have the form:

$$\tilde{P}(E) = A \cdot \exp\,(-\frac{E}{kT}) \tag{4.2}$$

This is however the distribution seen by the electrons within the plasma. Outsiders see this distribution boosted by the Energy X, the constant potential on which the plasma is sitting. (Remember that electrons within a plasmas don't see any potential.)

$$P(E) = \begin{cases} 0 & E < X \\ A \cdot \exp\,(-\frac{E-X}{kT}) & E > X \end{cases} \tag{4.3}$$

If the dump electrode is now lowered to V some electrons will escape over the confining barrier. In order to simplify my life I will only lower the barrier so far, that most electrons remain confined. This way the "self-potential" remains the same. The number of electrons escaping is the fraction with kinetic energy $T > (V - X)$. With a Boltzmann distribution this equals to:

$$\begin{aligned} N(V) &= B \cdot \int_{V-X}^{\infty} P(E)dE \\ &= B \cdot \exp\,(\frac{X}{kT}) \cdot (-kT) \exp\,(\frac{E}{kT})\,|_{E=V-X}^{\infty} \\ &= B \cdot kT \exp\,(\frac{2X}{kT}) \exp\,(-\frac{V}{kT}) \end{aligned} \tag{4.4}$$

All these electrons will fly to the Faraday-cups. On this path they will gain the kinetic energy $X + D$. The energy distribution on the Faraday cups will thus look the following:

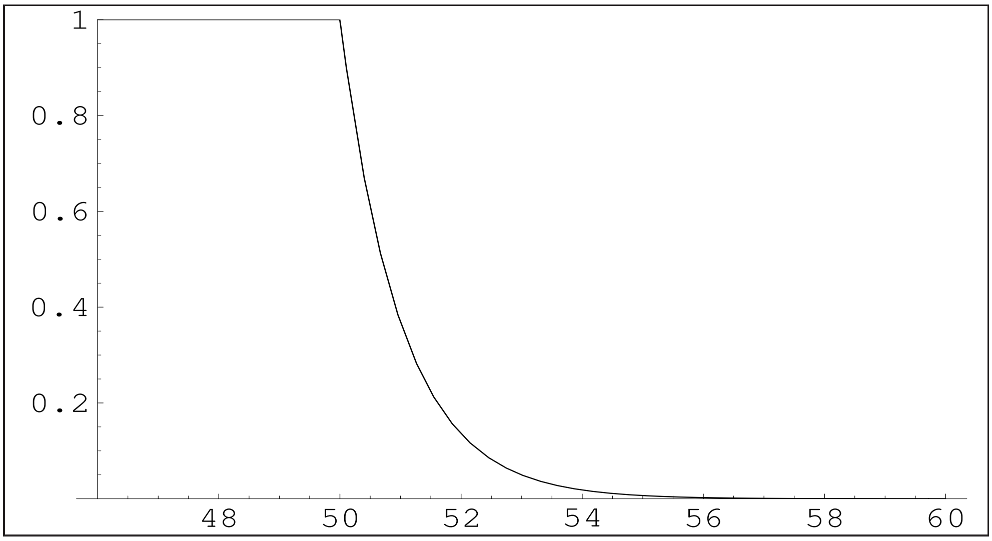


Figure 4.2: This distribution is strictly not true as the potential is lowered to much. It however describes the situation well for higher energies.

4.2 Measuring the temperature

The determination of the temperature is now quite clear. All you have to do is measure the number of electrons escaping over a lowered potential for different potentials and fit a Boltzmann-distribution to these data.
The tricky thing is only how to deduce the number of electrons out of the raw data. A great deal of signal is constant noise which can be subtracted from the signals as shown later.

4.2.1 Noise measurements

There are three sources of noise in MINERVA .

1. ADC-noise. The Incaas have some "intrinsic" noise. If the ADC's are suited well for their purpose, this noise will prove to be negligible.
2. Preamplifier-noise. The preamplifiers also have some noise on them
3. Pickup. The Faraday cups are sitting right next to electrodes, which have voltage changes of $\approx 50V$ in the short time of $10\mu s$. These signals are seen on the Faraday cups as a complex "oscillation"-pattern.

4.2.2 Noise measurements

ADC noise

The noise of the ADC's is easy to measure. All there is to do is measure nothing ($\Leftrightarrow 0V$). All variations in channels is due to internal noise.
This measurement not only shows the noise of the ADC's but also indicates, that

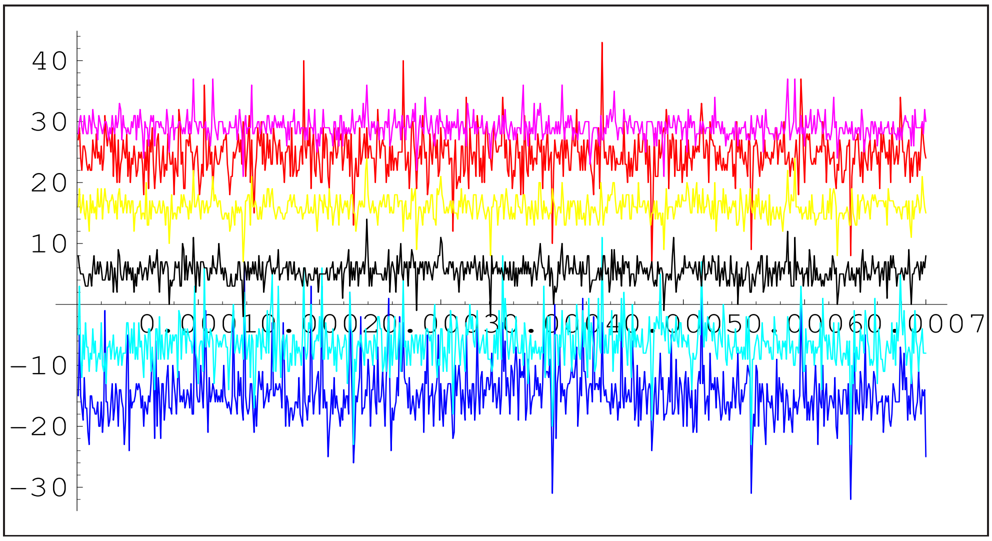


Figure 4.3: Noise measurement on the INCAA-ADC's. Remember that the range of the Incaas is -2047–2048.

the baselines of the different channels must be taken in account. The height of the baseline is determined by simply calculating the average. The noise is equivalent to the average deviation of the data. This is the normal definition for noise.

channel	1	2	3	4	5	6
noise[#]	3	3	3	2	2	2
baseline	-15	-6	24	29	16	5

System noise

The noise of the system is trickier to determine. Of course the main goal is to measure the noise in a situation, which is as "life-like" as possible. In order to do this I measured the noise for a puls from -100V to -50V several times. In the for me interesting range of about 50V the noise did not appear to depend on the exact voltage. It however varied for much larger or smaller pulses. In addition a reassured myself, that the pickup did not change for measurements with different hold times. Then I took the average of various pulses going from -100V to -50V confining voltage. This signal was then considered to be the "reference-signal". Now I compared a single measurement with this average. The difference is (if I measured the same event every time) the noise of the system.

The results of this measurement is shown in ***. This is the interesting noise, since

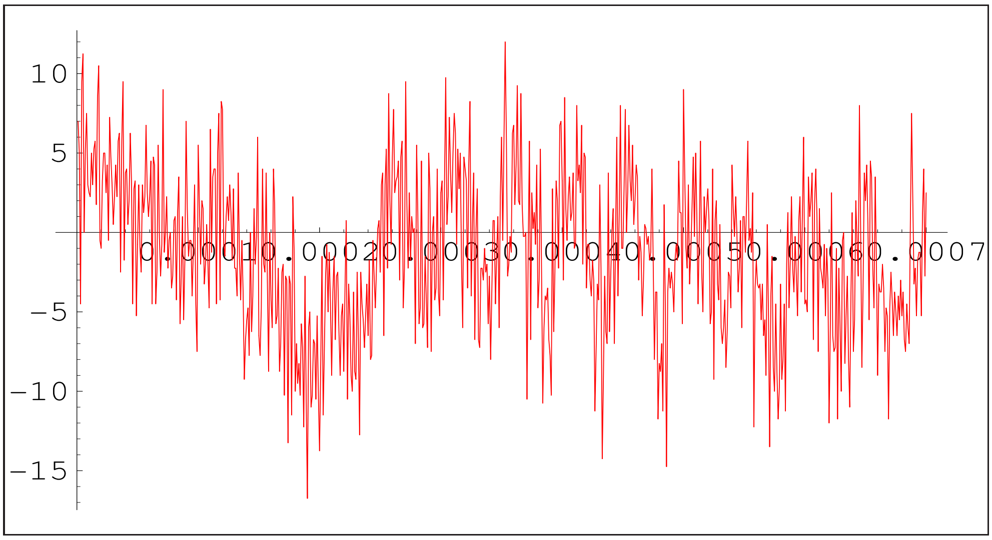


Figure 4.4: Noise measurement on the whole system. This time only the "noisiest" channel is shown. This is because all the signals lie much to close together.The first 20 bins were thrown away, since the Pickup is to strong in that region

it makes a statement for the whole system, with all devices attached.

channel	1	2	3	4	5	6
noise[#]	3	3	4	2	2	4

Pickup

As I have already mentioned the Faraday cups not only see electrons but also some electrical oscillations. These oscillations are supposingely from the puls that is applied to the dump electrode. A good argument herefor is that the pickup is mainly seen on the front electrode [L7].
In principle this phenomenon should be understandable. This is however not the case in reality. But as shown in the section on total system noise (***) the pickup is very repeatable. In addition to this, the pickup is nearly independent of the voltage in the typical measuring ranges [$V_{\text{top}} = -50V{-}{-}60V$]. This is why I feel safe to simply subtract a constant pickup signal, which I choose to be the average signal of the pickup-measurements I made.

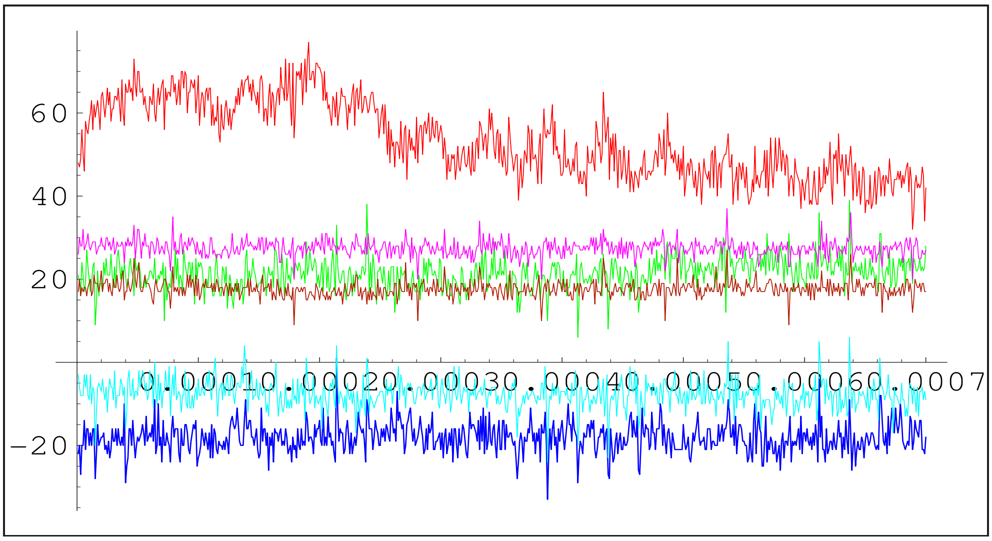


Figure 4.5: A typical "Pickup"-event. The processes that are involved are not understood well enough. However due to shot-to-shot reproducibility the pickup can be measured and subtracted.

4.2.3 Calibration of the system

Although I in theory don't have to measure the number of electrons on a Faraday cup because the interesting part of the Boltzmann-distribution doesn't depend on the exact number (It only appears in the constant) I am still interested in this. Calibrating the system gives a good cross-check for my measured data. In order to calibrate the system I inserted a capacitor in front of the preamplifier. If I then apply a voltage puls of given height, then the preamplifier will see exactly the same thing as a dump puls. The voltage puls can be transformed into a charge puls via $Q = V \cdot C$.

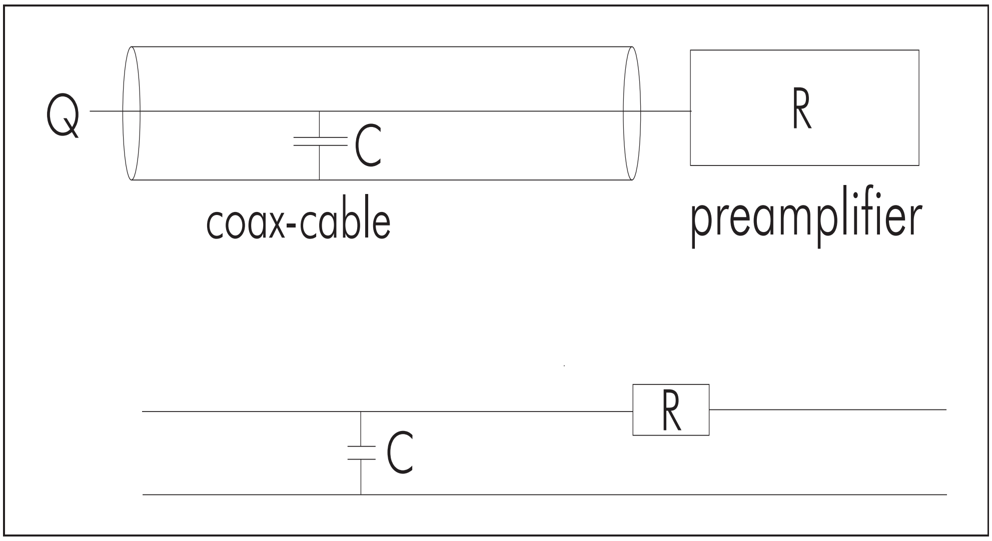


Figure 4.6: The detection system can be considered as a RC-Glied. When the electrons are dumped the charge is deposed on the capacitor and then flows of over the resistance.

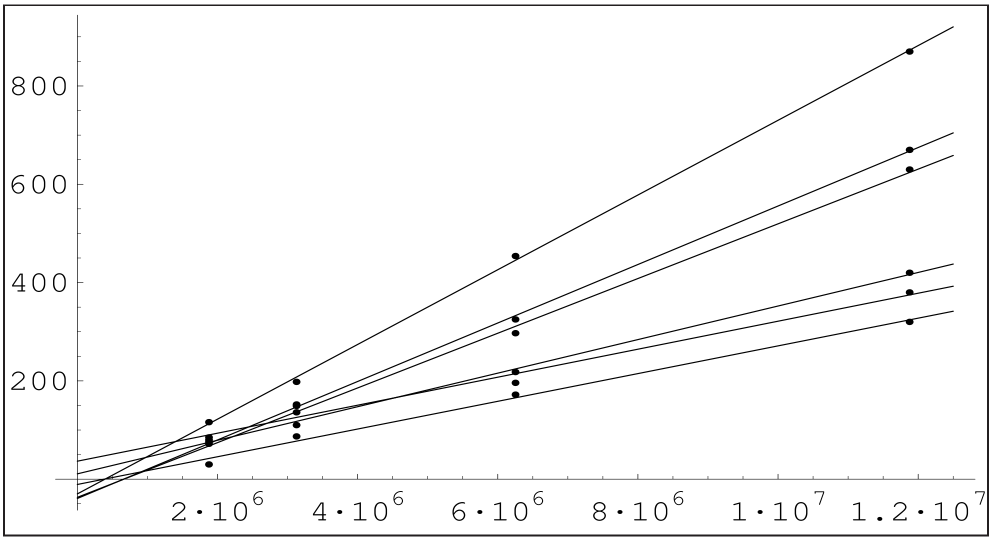


Figure 4.7: Calibration of the readout system.Due to different capacities of the signal cables the pitches of the preamplifiers are not equal.

The coefficients for this calibration are shown in ***:

detector	fit ADC→e^-	noise equivalent in e^-'s
1	$661500 + 16800 \cdot x$	50400
2	$672300 + 17900 \cdot x$	53700
3	$396700 + 13100 \cdot x$	52400
4	$299200 + 29200 \cdot x$	58400
5	$415200 + 35200 \cdot x$	70000
6	$-1139600 + 34400 \cdot x$	120000

4.2.4 Electron data

In this section I will present the actual data I took. Figure *** shows a raw signal stemming from the Farady cup P7.

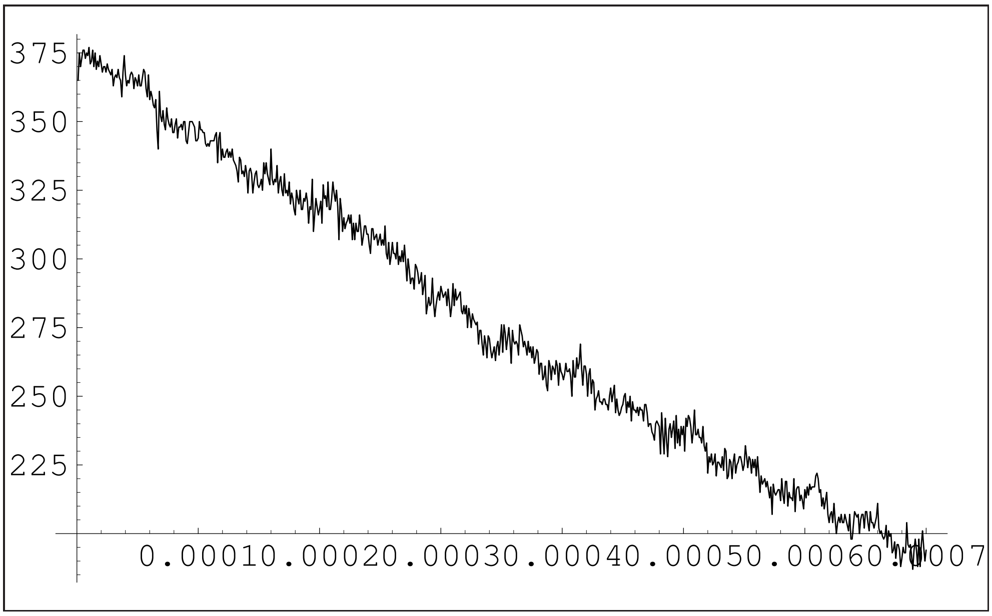


Figure 4.8: A raw event as seen on MINERVA . This is a signal on P7 when all the electrons were dumped

This data was first corrected by subtracting the pickup. The solid line denotes the exponential fit to the data.

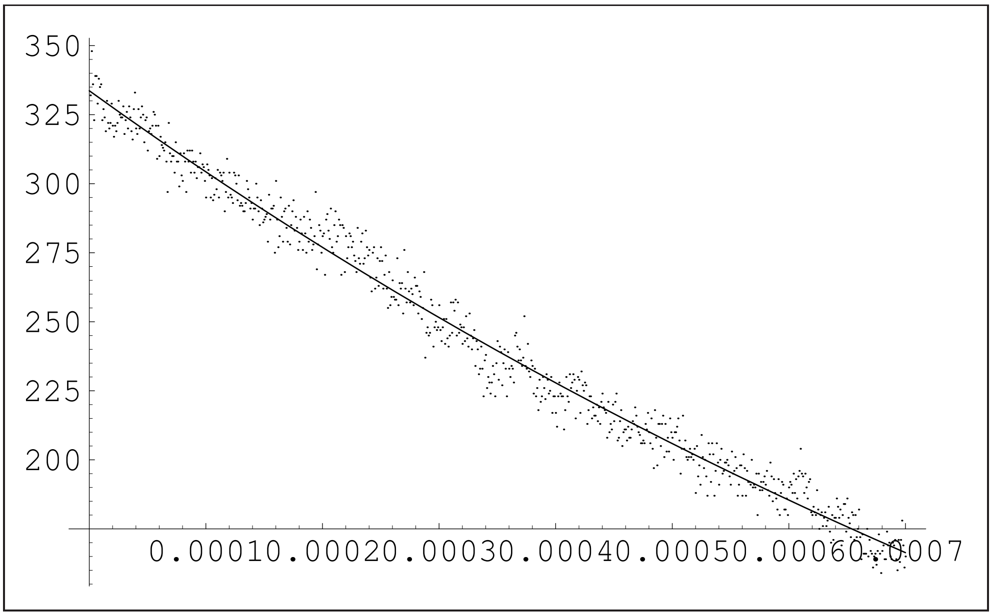


Figure 4.9: A corrected signal. P7 sees *** electrons.

To the data I fitted a function $V(t) = V_0 \cdot \exp\left(\frac{t}{\tau}\right)$ with the two fit parameters V_0 and τ. The question may arise whether a third fit parameter which states a constant offset has to be considered. The answer is no. Consider the pickup-signal. It can be written as $V_0(t) = \mathcal{O} + V_{\text{pickup}}(T)$. The signal one sees with electrons is given by $V_e(t) = V_0(t) + a \cdot \exp(-\frac{t}{RC})$. So by subtracting a pickup signal from the actual electron signal you automatically get rid of any constant offset. The fit was done by a MATHEMATICA-program using the χ^2-method.

4.3 Results

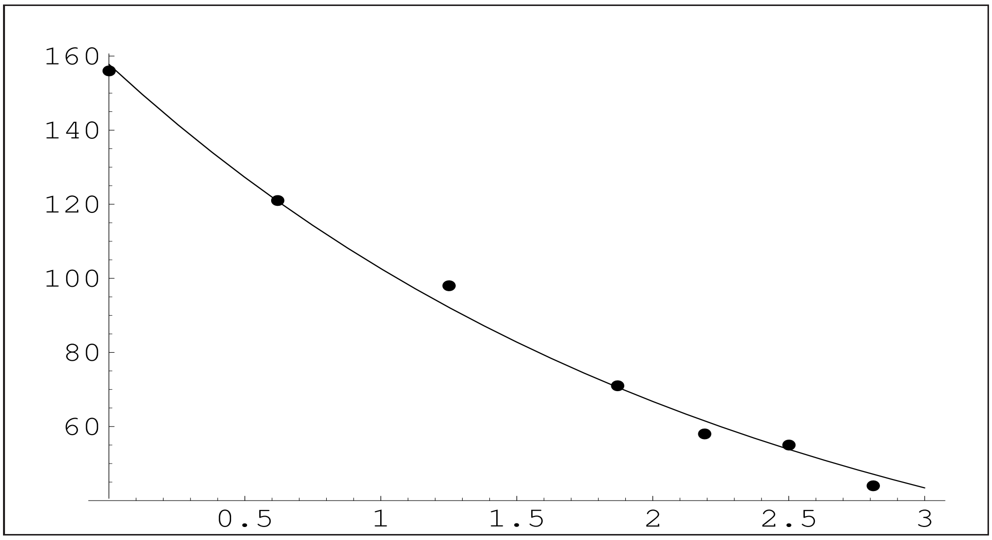


Figure 4.10: The measured energy distribution looks very much like a Boltzmann-distribution.

Fig. *** shows a typical temperature fit for one of the detectors. Clearly the energy distribution shows the Boltzmann-behaviour we would expect. This allows two statements:

- The plasma locally settles at a temperature. In addition the temperature measured by the different electrodes do not differ to much.
- The shot-to-shot reproducibility is fairly good. Since different points on the energy distribution are taken in separate shots this is not self-evident.

The typical error for a temperature is in the 5–10% region. It is clear that due to the noise, collectors with large signals give a more accurate temperature than the ones with small signals. I therefor calculate a weighted mean value for the overall temperature using:

$$
\begin{aligned}
w_i &= \frac{1}{\sigma_i^2} \\
\overline{T} &= \frac{\sum_{i=1}^{6} w_i T_i}{\sum_{i=1}^{6} w_i}
\end{aligned}
$$

Using this I calculate the following temperature-evolution:

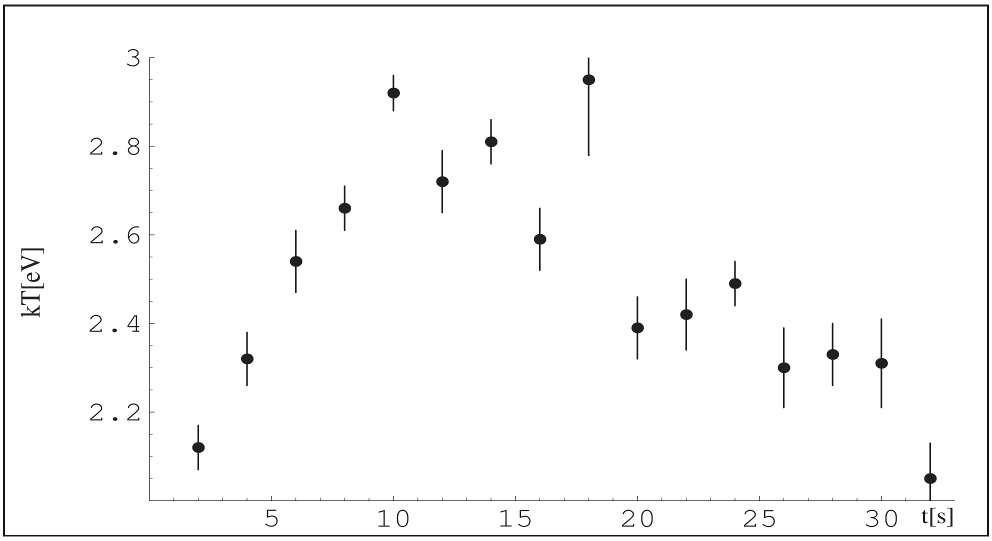


Figure 4.11: Unfortunately the MINERVA plasma does not show the synchrotron radiation cooling characteristic.

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